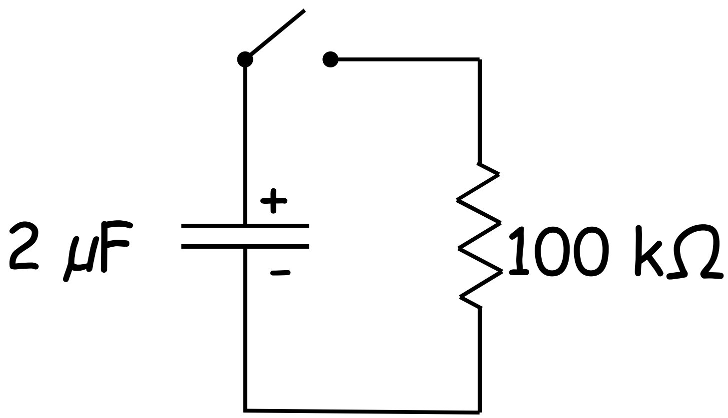
RC Circuits

Text section 28.4

Practice: Chapter 28,

Objective Question 7
Conceptual Question 6
Problems 37, 41, 43, 63

Discussion



The capacitor has an initial charge of 20 μC.

a) The switch is now closed : What is the <u>initial</u> current through the resistor?

b) How long would it take for all the charge to leak off if the current stayed constant?

Result:

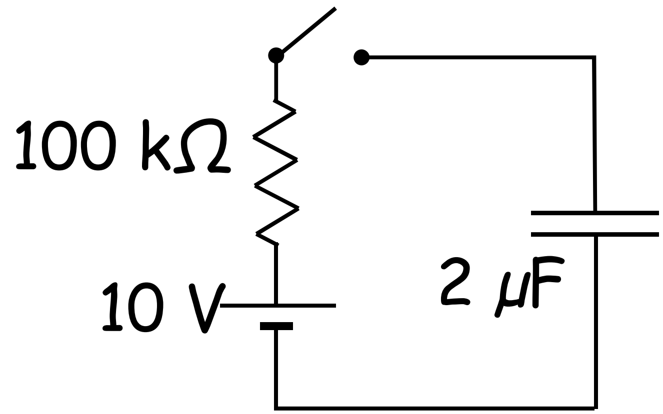
time = resistance x capacitance
(the units work out to seconds).

Actually, the current is not constant; as the charge decreases, the voltage decreases, so the current decreases, so charge decreases more and more slowly ...

Even so, (resistance) x (capacitance) is a good first estimate of the time.

Quiz

The capacitor has zero initial charge. What is the battery current immediately after the switch is closed?



A) 20 μA

B) 5 μA

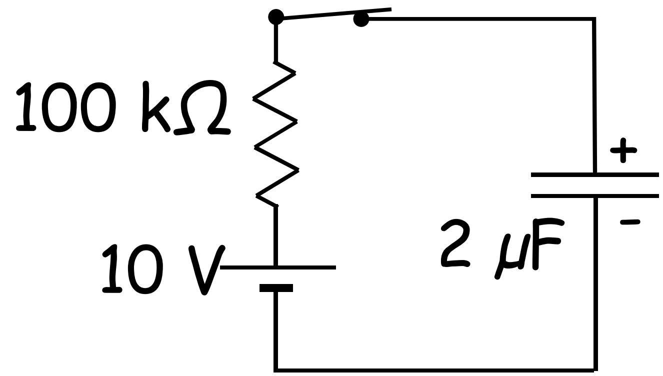
C) 50A

D) 100 μA

E) 200 mA

Quiz

The capacitor has zero initial charge. What will the charge on the capacitor be a long time after the switch is closed?



A) 20 μC

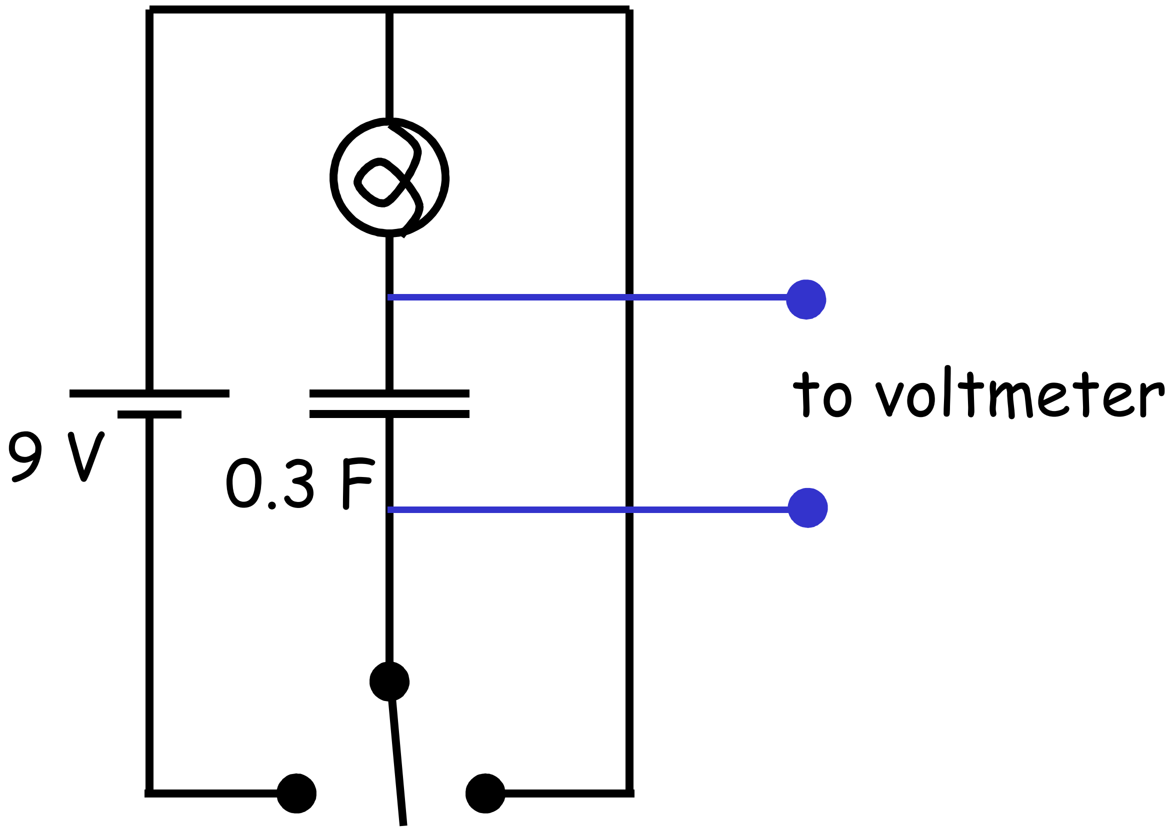
B) 5 μC

C) 2 C

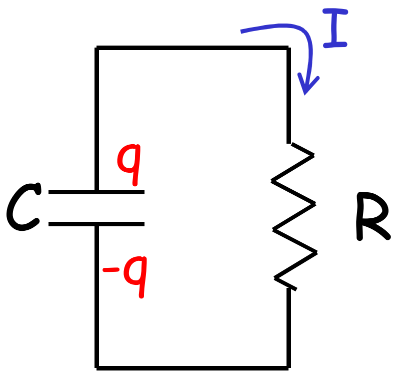
D) zero

E) It will increase without limit

Demonstration



Discharging a Capacitor



Given: R, C, q_o (initial charge)

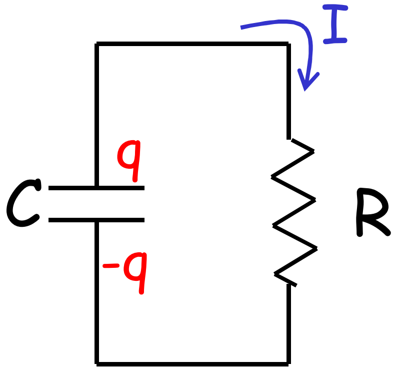
Find: $q(t)$ and $I(t)$

1) $$\frac{q}{C} - IR = 0$$ *(Kirchhoff's Loop Rule)*

2) $$I = -\frac{dq}{dt}$$ *(- sign because q **decreases** for I > 0)*

$$\Rightarrow \quad \frac{q}{C} = -R\frac{dq}{dt}$$

$$\frac{dq}{dt} = -\frac{q}{RC}$$ where $q = q(t)$, $q(0) = q_o$



$$\frac{dq}{dt} = -\frac{q}{RC}$$

where $q = q(t)$

$$q(0) = q_o$$

*This is a **differential equation** for the **function** q(t), subject to the **initial condition** $q(0) = q_0$.*

We are looking for a function which is proportional to its own first derivative.

To solve:

$$dq = -\frac{q}{RC}dt$$

$$\Rightarrow \qquad \frac{dq}{q} = -\frac{1}{RC}dt$$

bring all the "q" variables to one side, all the "t" variables to the other

$$\int_{q=q_o}^{q(t)} \frac{dq}{q} = \int_{t=0}^{t} (-\frac{1}{RC})dt$$

then integrate each side

$$\underbrace{\ln q - \ln q_o}_{\ln(\frac{q}{q_o})} = -\frac{t}{RC}$$

$$\Rightarrow \qquad \boxed{q(t) = q_o e^{-\frac{t}{RC}}}$$

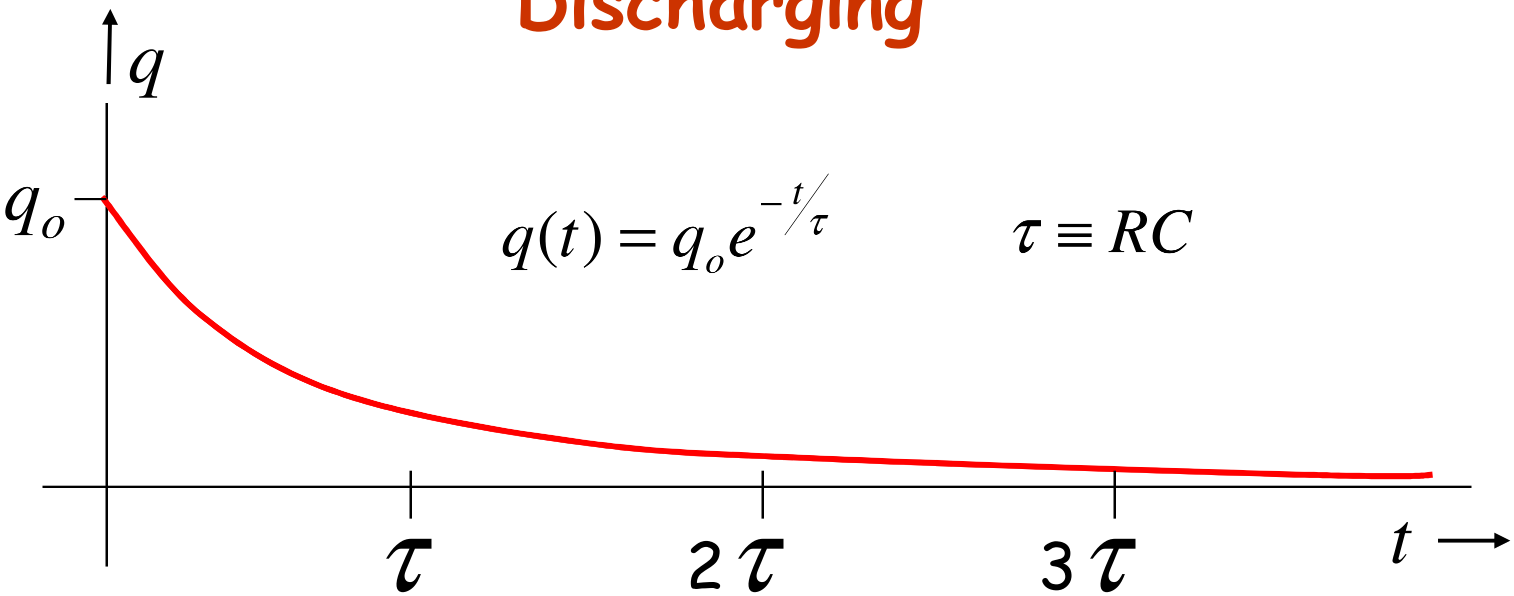
RC is called the "time constant" or "characteristic time" of the circuit.

Units: 1 Ω x 1 F = 1 second *(Proof: exercise)*

Write τ ("tau") = ***RC***, then

$$q(t) = q_o e^{-\left(\frac{t}{\tau}\right)} \quad \text{(discharging).}$$

Discharging



$$q(t) = q_o e^{-t/\tau} \qquad \tau \equiv RC$$

$t = \tau$, $\quad q \approx 0.37\, q_o$

$t = 2\tau$, $\quad q \approx 0.14\, q_o$

$t = 3\tau$, $\quad q \approx 0.05\, q_o$

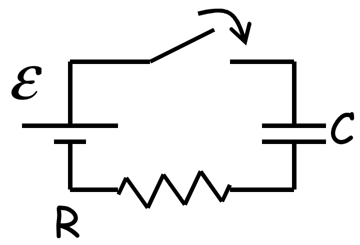
$t \rightarrow \infty$, $\quad q \rightarrow 0$

Quiz

A capacitor is charged up to 18 volts, and then connected across a resistor. After 10 seconds, the capacitor voltage has fallen to 12 volts. What will the voltage be after another 10 seconds (20 seconds total)?

A) 0
B) 6V
C) 8V
D) 9V
E) 10V

Charging



C is initially uncharged, and the switch is closed at t=0. After a long time, the capacitor has charge Q_f.

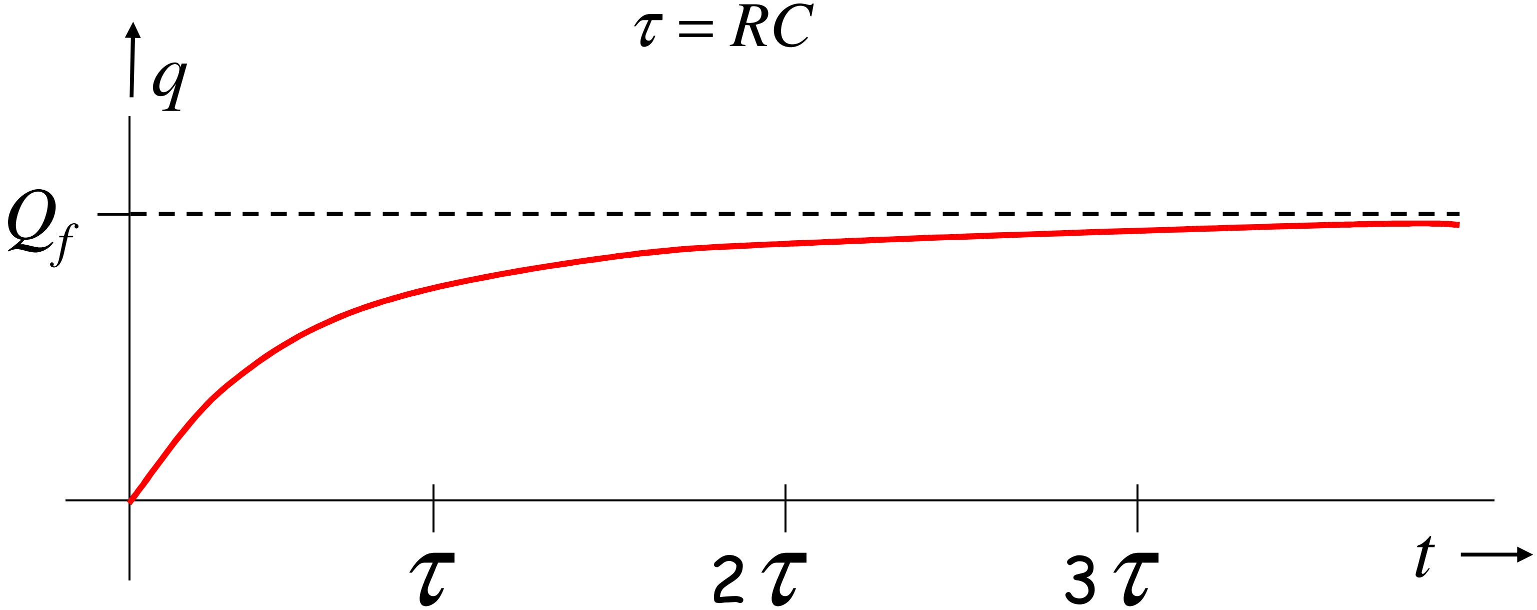
Then, $$q(t) = Q_f\left(1 - e^{-t/\tau}\right)$$ where $\tau = RC$.

Question: What is Q_f equal to?

Charging

$$q(t) = Q_f\left(1 - e^{-t/\tau}\right)$$

$$\tau = RC$$



$t = 0$, $q=0$

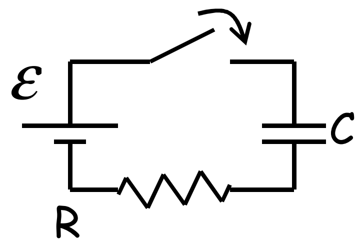
$t = RC$, $q \approx 0.63\ Q_f$

$t = 2\ RC$, $q \approx 0.86\ Q_f$

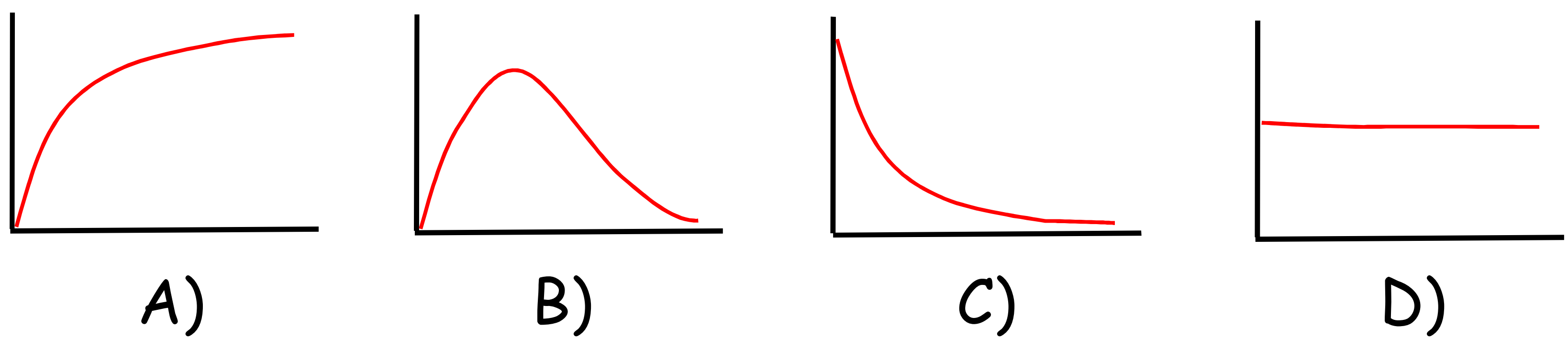
$t = 3\ RC$, $q \approx 0.95\ Q_f$

etc.

Quiz

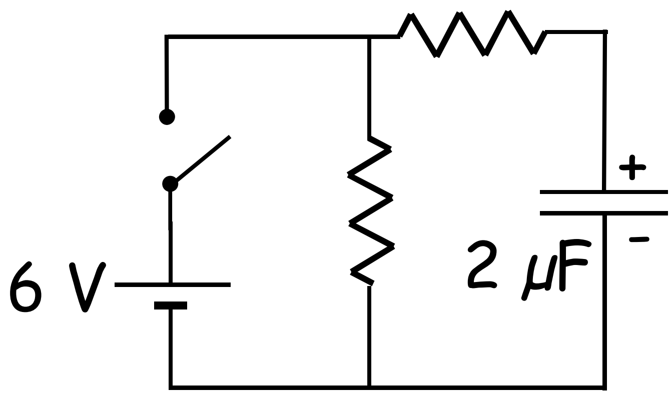


C is initially uncharged, and the switch is closed at t=0. Which graph below shows the ***current*** *as a function of time?*



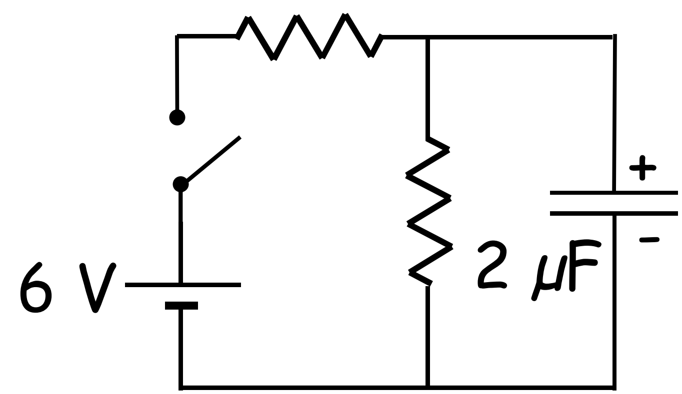
Exercise

The capacitor has zero initial charge, and each resistor is 200 kΩ. *Find the charge on the capacitor, and the current through each component, as functions of time.*



More Difficult Exercise

The capacitor has zero initial charge, and each resistor is 200 kΩ. *Find the charge on the capacitor, and the current through each component, as functions of time.*



"RC" Circuits

- a capacitor takes *time* to charge or discharge through a resistor

- "time constant" or "characteristic time"

$$\tau = RC$$

(1 ohm) x (1 farad) = 1 second