The Biot-Savart Law

(Text section 30.1, 30.2)

Practice: Chapter 30,
Objective Questions 4, 5, 9
Conceptual Questions 1, 11
Problems 7, 9, 11, 19, 65

Sources of $\vec{B}$

B exerts a force on moving charges.

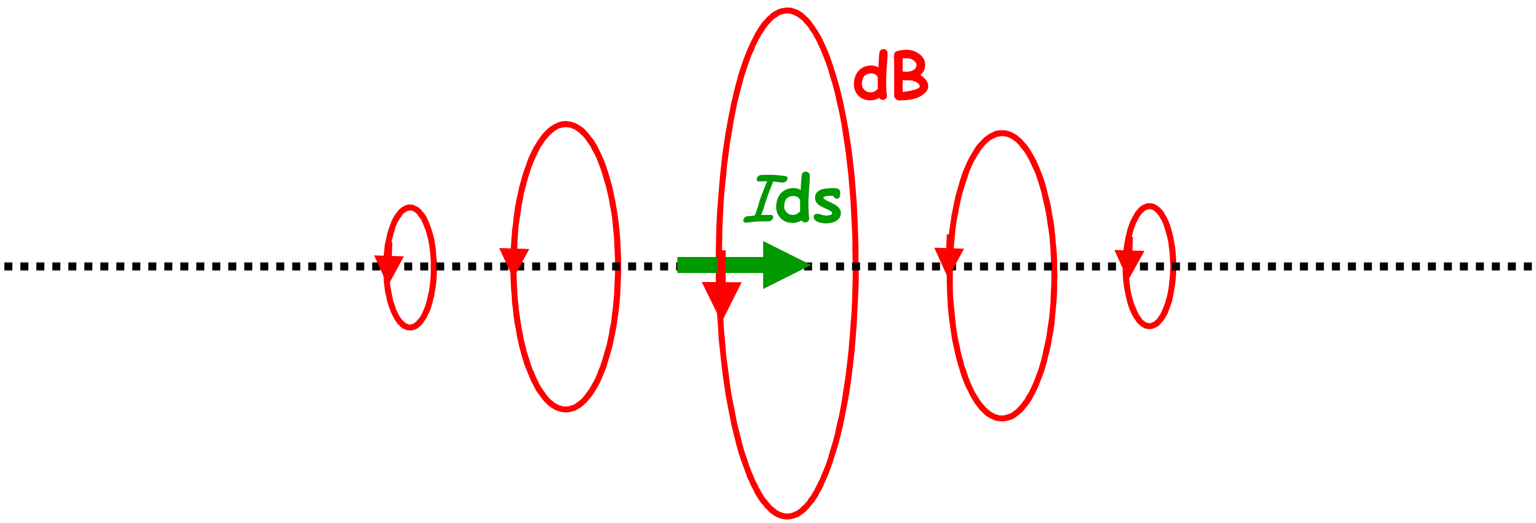
But also, moving charges create magnetic fields.

Two equivalent ways of calculating **B** produced by currents:

i) Biot-Savart Law: Field of a "current element" (analagous to a point charge in electrostatics).

ii) Ampère's Law: An integral theorem.

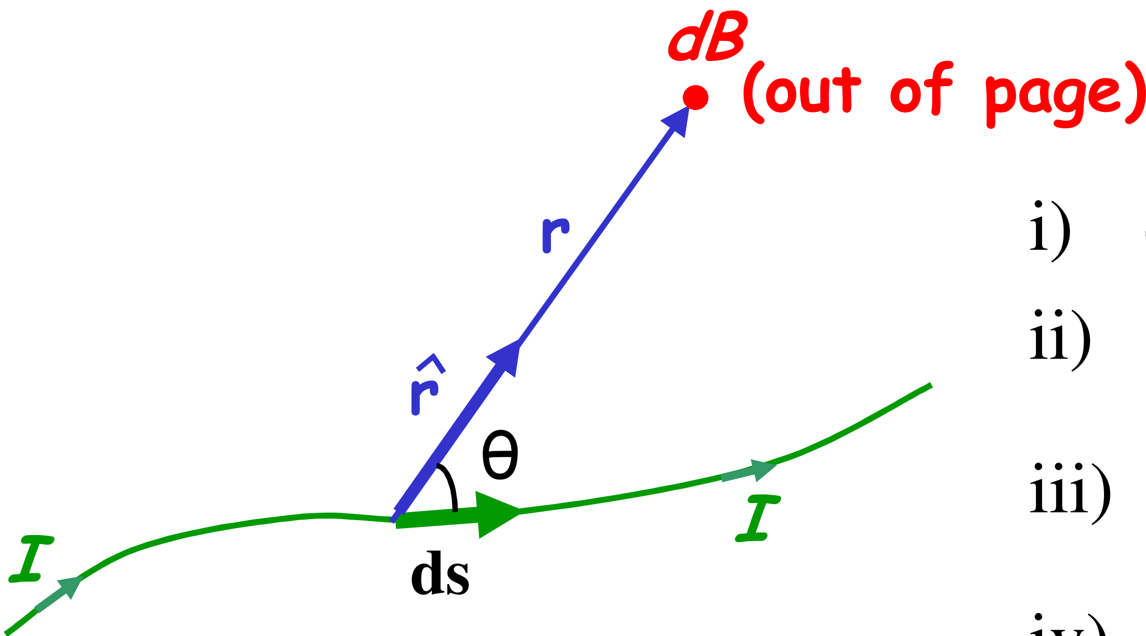
Biot and Savart: each "current element" I **ds** *(a very short length ds of wire, carrying current I) produces a field* **dB** *throughout space:*



In reality, the current element is part of a complete circuit, and only the ***total*** *field due to the* ***entire circuit*** *can be observed.*

Biot-Savart Law

Field produced at a vector distance **r** *by a "current element"* **I ds** :

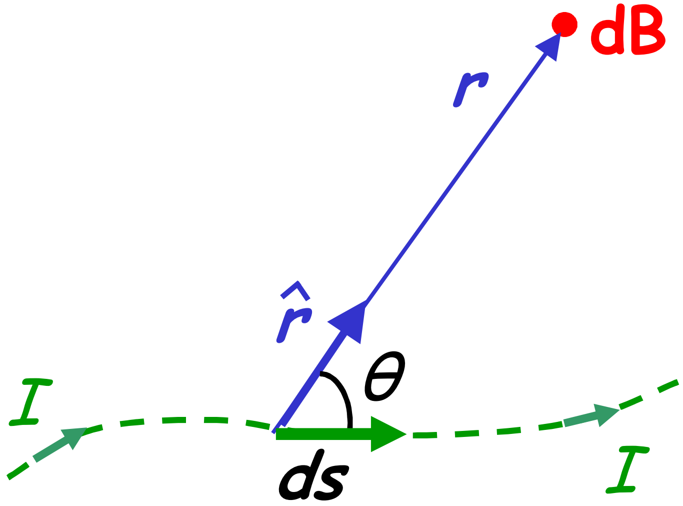


i) $\mathbf{dB} \perp$ current

ii) $\mathbf{dB} \perp \mathbf{r}$

iii) $|\mathbf{dB}| \propto \dfrac{1}{r^2}$

iv) $|\mathbf{dB}| \propto \sin\theta$



$$\mathbf{dB} = \frac{\mu_o}{4\pi} \frac{I\mathbf{ds} \times \hat{\mathbf{r}}}{r^2}$$

$$\mu_o = 4\pi \times 10^{-7} \ \ \mathrm{T \cdot m/A}$$

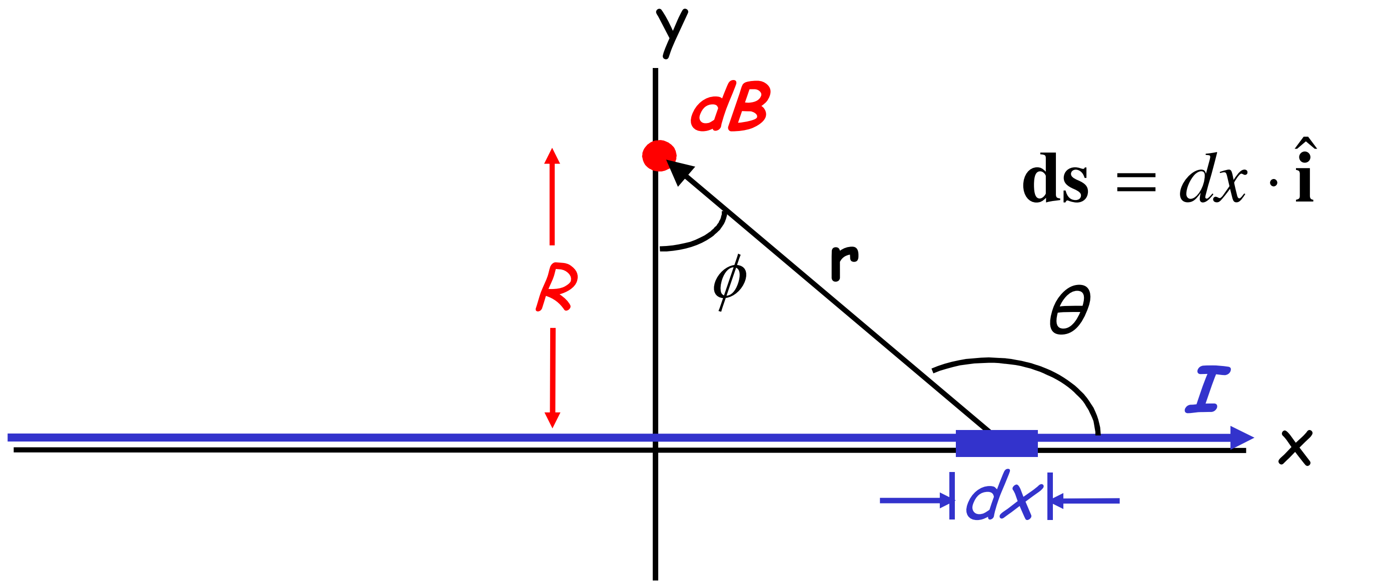
("permeability of vacuum")

We get the total **B** by integrating along the wire:

$$\mathbf{B} = \frac{\mu_o I}{4\pi} \int \frac{\mathbf{ds} \times \hat{\mathbf{r}}}{r^2}$$

Example:

Find **B** at a distance R from a long straight wire

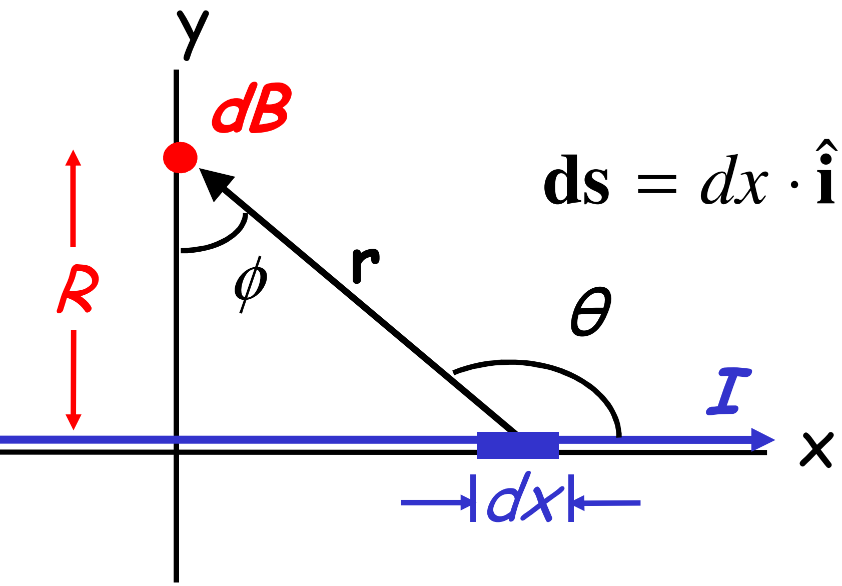


$$dB = \frac{\mu_o I}{4\pi} \frac{\sin \theta}{r^2} dx \quad \text{(in z - direction)}$$

(note $\sin\theta = R/r = \cos\phi$)

Quiz

The magnitude of the field will be



A) $B = \dfrac{\mu_0 I}{4\pi} \displaystyle\int_{-\infty}^{\infty} \dfrac{dx}{R^2 + x^2}$

B) $B = \dfrac{\mu_0 I}{4\pi} \displaystyle\int_{-\infty}^{\infty} \dfrac{R\,dx}{(R^2 + x^2)^{3/2}}$

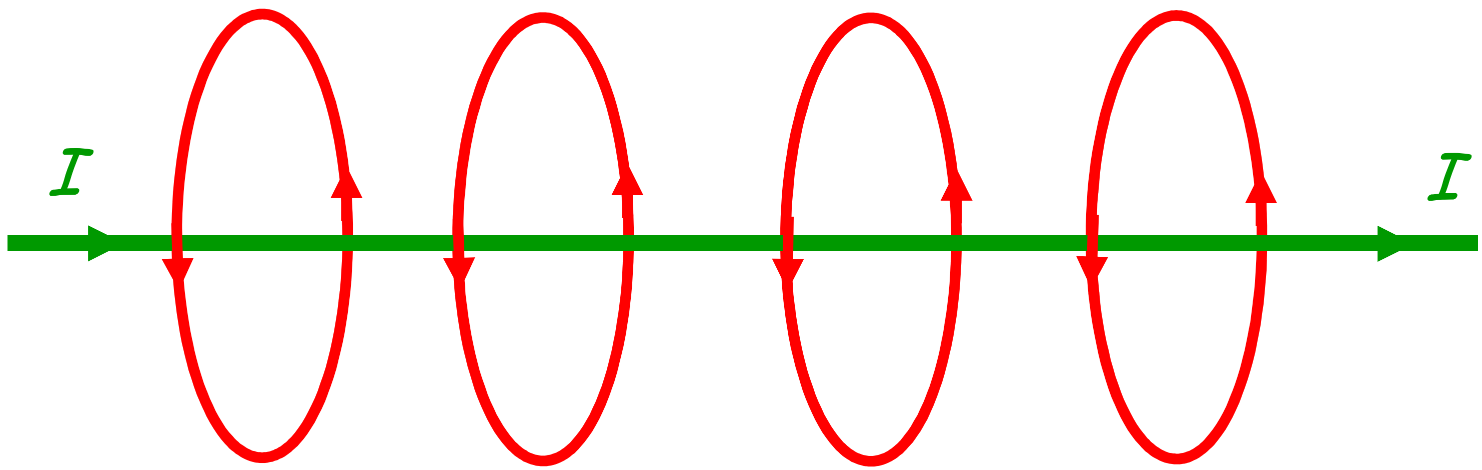
C) $B = \dfrac{\mu_0 I}{4\pi} \displaystyle\int_{-\infty}^{\infty} \dfrac{x\,dx}{(R^2 + x^2)^{3/2}}$

D) None of the above

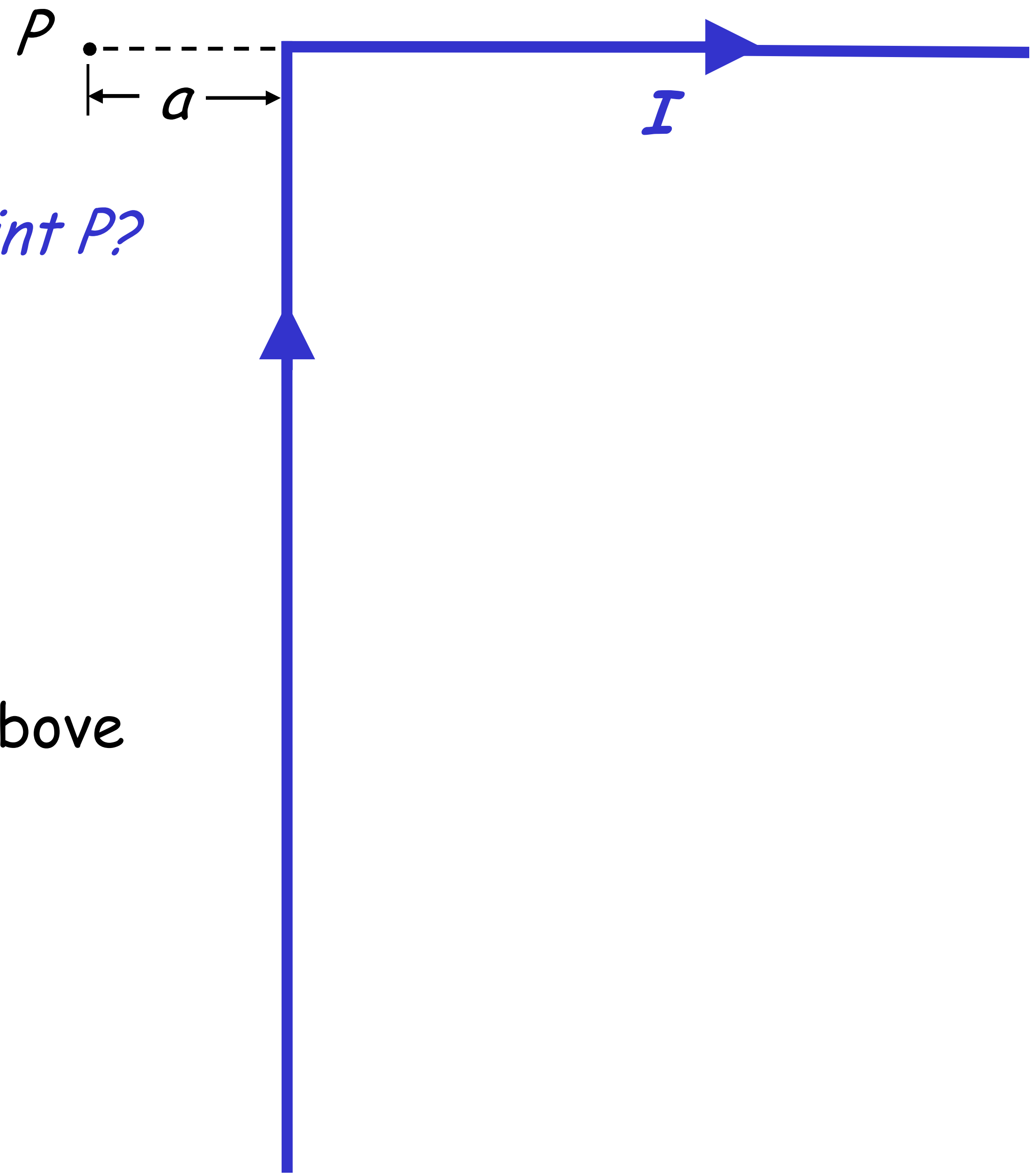
Result: the field produced at a distance R from a long straight wire is

$$B = \frac{\mu_0 I}{2\pi R}$$

The field lines form circles around the wire. Note the right-hand rule.



Quiz



What is the field at point P?

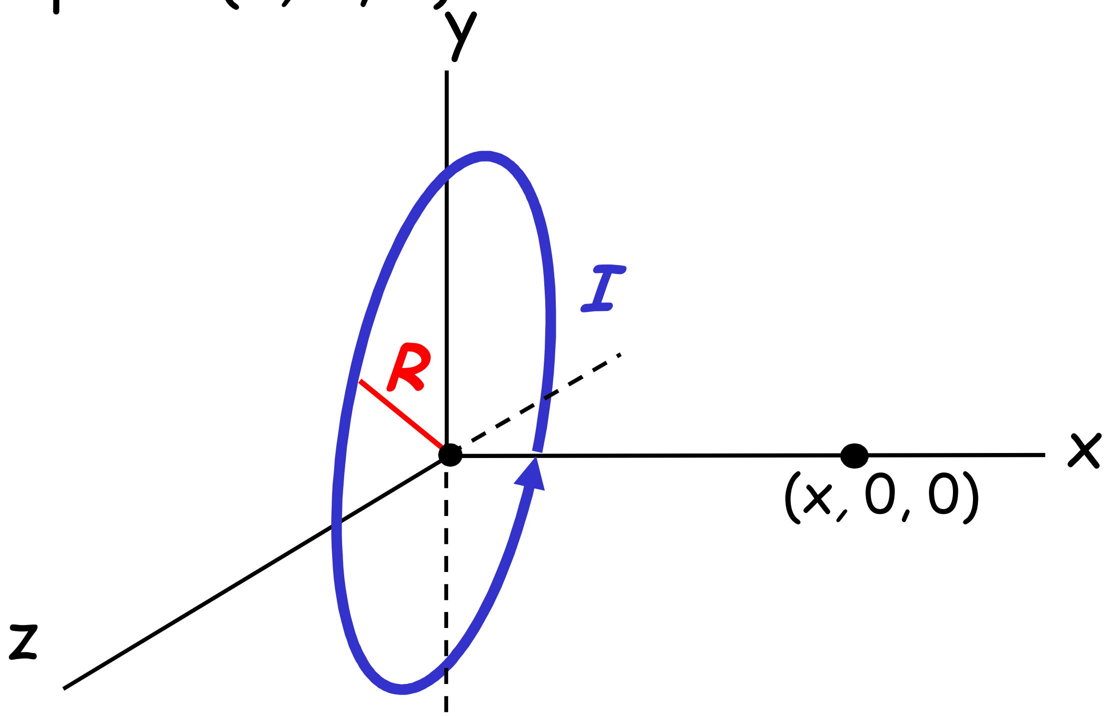
A) $\mu_0 I/(4\pi a)$
B) $\mu_0 I/(2\pi a)$
C) $\mu_0 I/(\pi a)$
D) *zero*
E) none of the above

Example

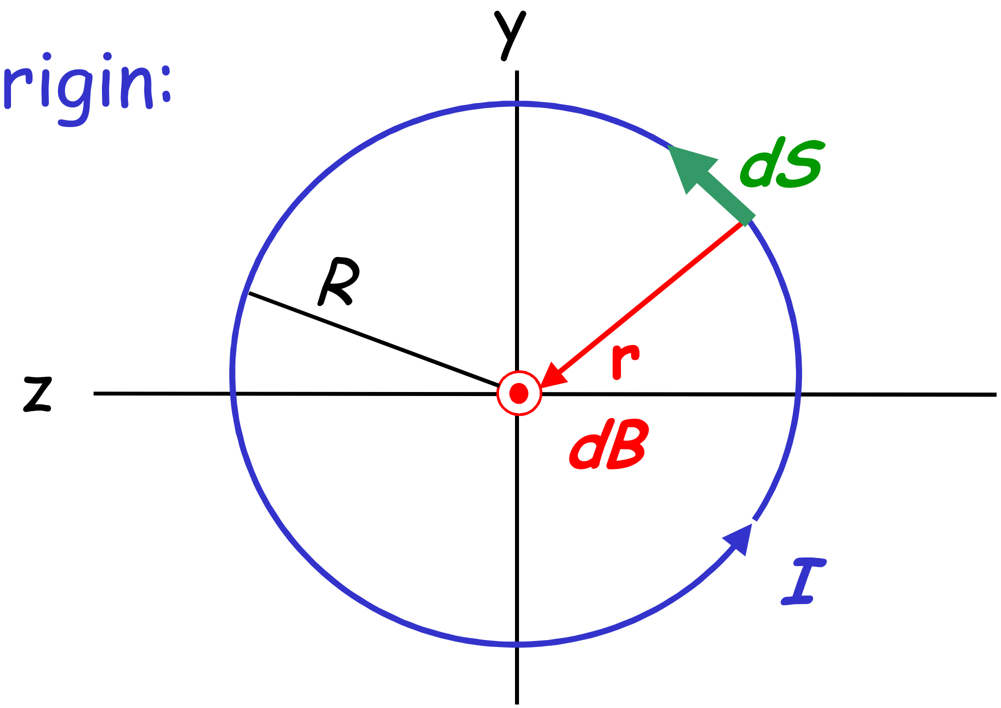
Circular loop in y-z plane. Find **B**.

a) at the origin

b) at point (x, 0, 0)

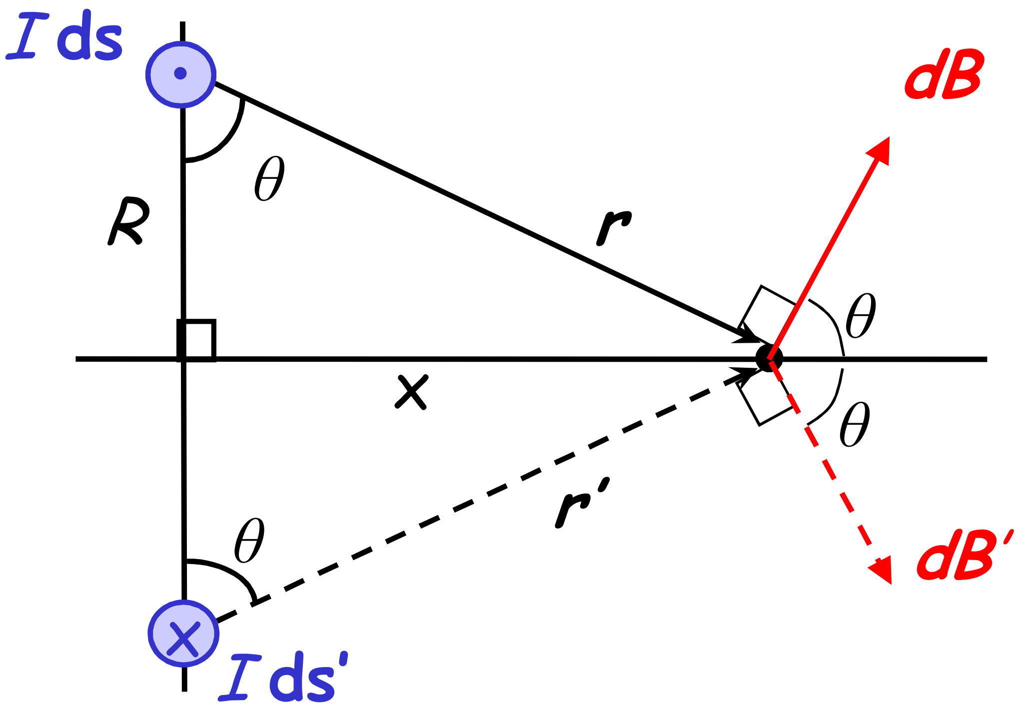


At the Origin:



At the point (x,0,0):

Pick a short segment **ds** where the wire crosses the y – axis:



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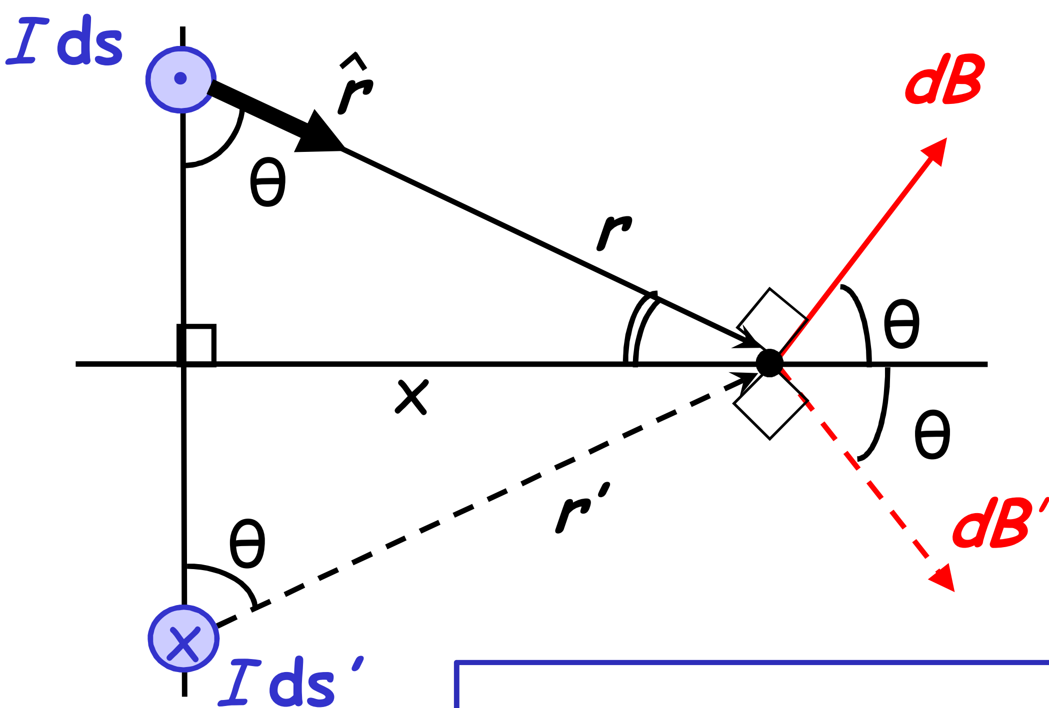


$$\left|\overrightarrow{\mathbf{dB}}\right| = \frac{\mu_o I}{4\pi} \cdot \frac{ds}{r^2}$$

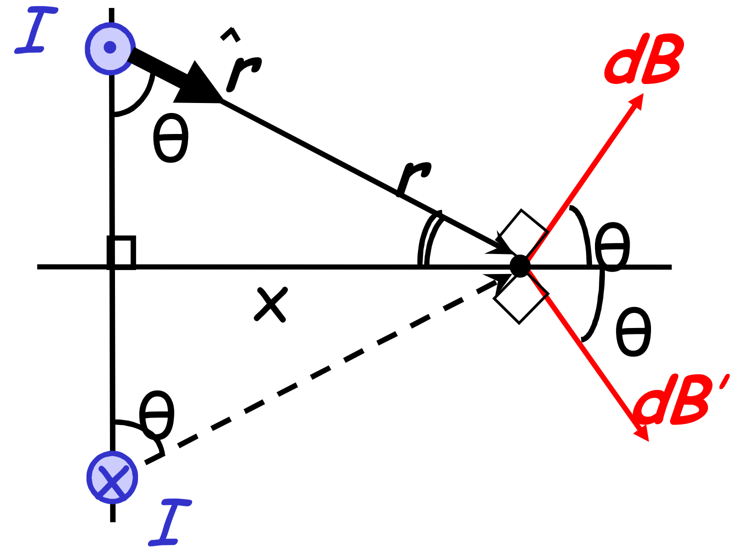
$$dB_x = \frac{\mu_o I}{4\pi} \cdot \frac{ds}{r^2} \times \cos\theta$$

The total field (x-component) is $B_x = \int \frac{\mu_o I}{4\pi} \cdot \frac{\cos\theta}{r^2} ds$

Quiz

$$B_x = \int \frac{\mu_o I}{4\pi} \cdot \frac{\cos\theta}{r^2} ds$$

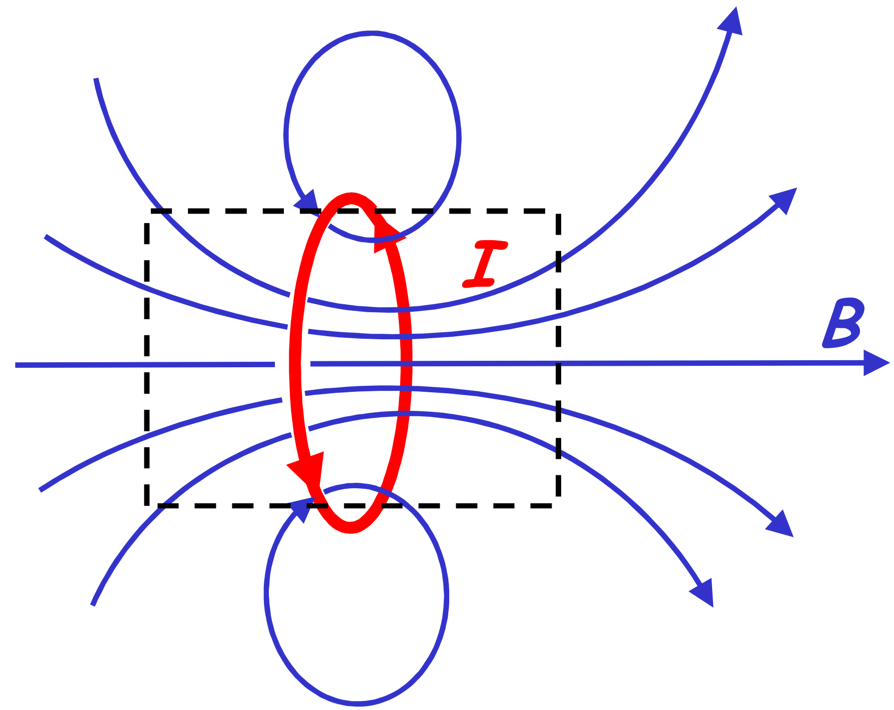
*Which quantities can be treated as **constant** when doing the integral above?*



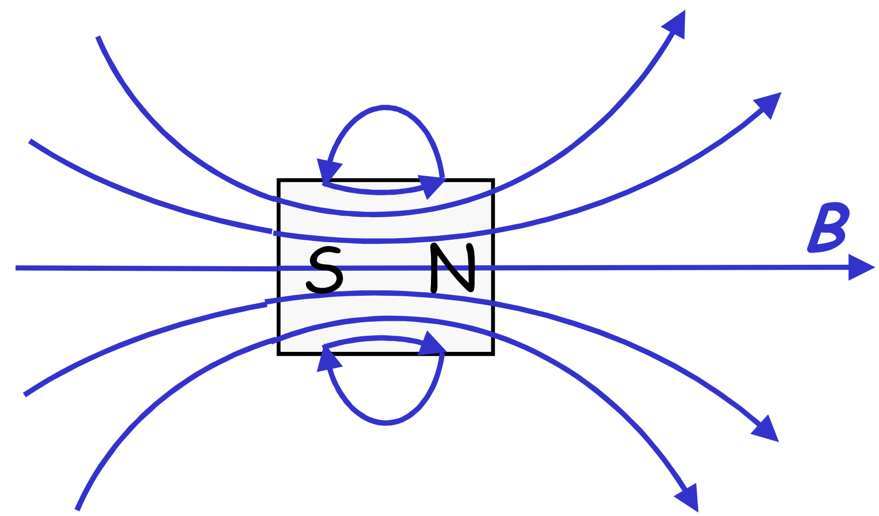
A) I
B) r
C) θ
D) all of the above
E) two of the above

Field patterns:

current loop



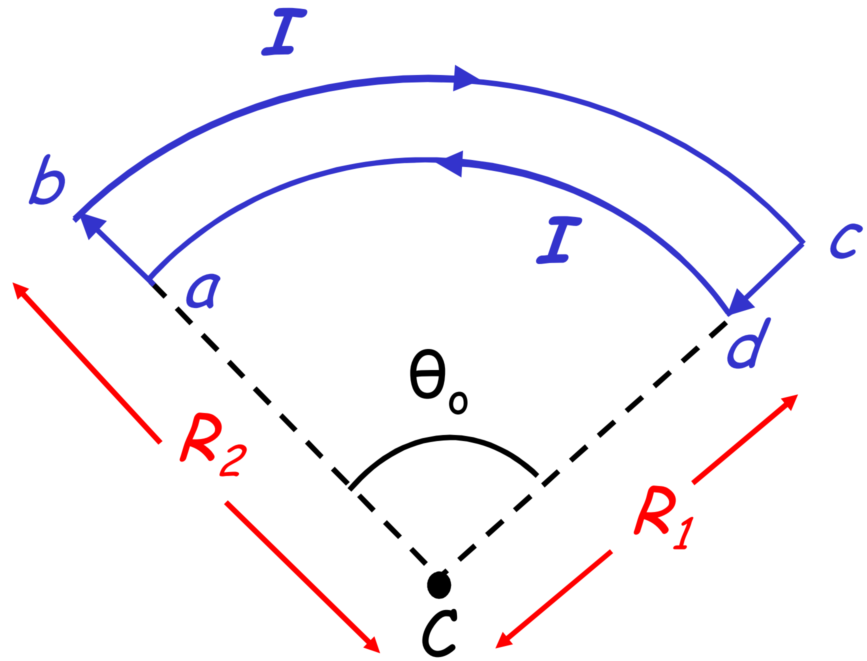
bar magnet



Example Problem

1) For each segment (*ab*, *bc*, *cd*, and *da*) state whether the field it contributes at point *C*:

i) points into the page
ii) points out of the page
iii) is zero.



2) Which field contribution is largest in magnitude?

3) Calculate the field at *C*.

Biot, Savart, and Newton's Third Law

A paradox?

Find the magnetic force each short current element exerts on the other, using the Biot-Savart picture: