STELLAR EVOLUTION CODE for MODEST

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Variables

Independent variables: m **(mass),** t **(time)**

Dependent variables:

r (radial distance)

p (pressure)

T (temperature)

L (energy flow rate)

Y_j (elemental number fraction) $\quad 1 \leq j \leq J$

J+4 dependent variables $\Longrightarrow$ **J+4 equations**

Structure equations

$$\frac{d}{dm}\frac{4\pi r^3}{3} = \frac{1}{\rho}$$

$$\frac{dp}{dm} = -\frac{Gm}{4\pi r^4}$$

Change of variables:

$$x = r^2, \qquad \mu = m^{2/3}$$

$$\Downarrow$$

$$dx = \frac{3}{4\pi\rho}\left(\frac{\mu}{x}\right)^{1/2} d\mu$$

$$d\log p = -\frac{3G}{8\pi p}\left(\frac{\mu}{x}\right)^{2} d\mu$$

At $x = \mu = 0$:

$$\frac{\mu}{x} \to \left(\frac{4\pi\rho}{3}\right)^{2/3}$$

Evolution equations

Energy balance equation

$$\dot{u} + p\left(\frac{1}{\rho}\right)^{\cdot} = q - q_\nu - \frac{dL}{dm}$$

Radiative regions - Diffusion approximation

$$L = -4\pi r^2 \frac{c}{\kappa\rho}\frac{dp_R}{dr} \quad , \qquad p_R = \frac{1}{3}aT^4$$

$$+$$

hydrostatic equilibrium

$$\Downarrow$$

$$d\log T = \nabla_R d\log p \quad , \qquad \nabla_R = \frac{\kappa L}{4\pi c G m}\frac{p}{4p_R}$$

Convective regions - Mixing length recipe

$$d\log T = \nabla d\log p, \qquad \nabla = f(r, L, m, p, T, Y)$$

Rate equations

$$\dot{Y}_j = R_j - \frac{dF_j}{dm} \quad , \qquad F_j = -K\left(\frac{\nabla_R - \nabla_A}{\nabla_R}\right)^2 \frac{dY_j}{dm}$$

Boundary conditions

Required: **$2\,J + 4$ conditions**

Centre ($m = 0$)

$$r = L = F_j = 0$$

$J + 2$ conditions

Surface (photosphere)

$$F_j = 0 \quad , \qquad L = 4\pi r^2 \sigma T^4$$

$$(\kappa p_G)_s = (1 - \Gamma_s)\tau_s g_s$$

where

$$\Gamma_s = \frac{\kappa_s L_s}{4\pi c G M}, \qquad \tau_s \approx 1$$

$J + 2$ conditions

Set of equations

$$dx = \frac{3}{4\pi\rho}\left(\frac{\mu}{x}\right)^{1/2} d\mu \quad , \qquad x_c = 0$$

$$d\log p = -\frac{3G}{8\pi p}\left(\frac{\mu}{x}\right)^{2} d\mu \quad , \qquad (\kappa p_G)_s = (1-\Gamma_s)\tau_s g_s$$

$$d\log T = \nabla d\log p \quad , \qquad L_s = 4\pi r_s^2 \sigma T_s^4$$

$$dL = (q - q_\nu - \dot{u} - p\dot{v})\, dm \quad , \qquad [v = 1/\rho], \quad L_c = 0$$

$$dF_j = \left(R_j - \dot{Y}_j\right) dm \quad , \qquad (F_j)_c = (F_j)_s = 0$$

Adaptive zoning

Differential eqns. $\Longrightarrow$ difference eqns.

$$i = 1, \ldots, N$$

Additional eqn. for mass m distribution:

$$d^2 f(r, L, m, p, T, Y) = 0$$

$$f = (m/M)^{2/3} - c \log p$$

Additional (2) boundary conditions:

$$m_c = 0 \quad , \qquad m_s = M \quad \text{[fixed mass]}$$

or

$$m_s = m_s^0 + \delta t \dot{M}(r_s, L_s, m_s) \quad \text{[mass loss]}$$

Solution

Difference scheme $\Longrightarrow$ **in difference equations:**

$$\dot{u}_i \rightarrow \frac{u(m_i) - u^0(m_i)}{\delta t}$$

Adaptive zoning $\Longrightarrow$ **in iterations:**

$$-u^0(m_i^{n+1}) = -u^0(m_i^n) - \frac{du^0}{dm}(m_i^{n+1} - m_i^n)$$

Newton-Raphson $\Longrightarrow$ **linear system (band matrix)**

Avoid singularity $\Longrightarrow$ **in convective diffusion terms:**

$$K\delta t/\Delta m^2 \leq 10^6$$

Input physics

• Equation of state: Fermi-Dirac integrals evaluated by Gauss-Laguerre quadrature. Provision for

- pair creation
- iron photodisintegration
- exchange and Thomas-Fermi corrections
- liquid/solid Coulomb corrections (De Witt et al.)
- H/He ionization
- H_2 formation

• OPAL and low temperature Alexander/Ferguson or (optional) Neuforge opacities.

• Active elements: H, He, C, N, O, Mg.(Mg stands for all isotopes with $Z > 12$)

• Nuclear energy rates according to Fowler/Caughlan.

• Neutrino losses: Itoh et al. 1995, optional recombination ν losses.

Initial configuration

Values of 11 vectors of dimension N:

$$\{r_i, L_i, m_i, p_i, T_i, Y_i(H), Y_i(He), Y_i(C), Y_i(N), Y_i(O), Y_i(Mg)\}$$

$$i = 1, \ldots, N$$

Option: uniform (the same for all i), of given $X(He)$ and Z

$$X(H) = 1 - X(He) - Z$$

Z break-down $\Longrightarrow$ Anders-Grevesse solar abundance.

Options:

- Change number of grid points N.
- Multiply all r_i, or all m_i, or all T_i, by constant factors.

'Non-canonical' models

Standard structure eqns. $\Longrightarrow$ quasi-dynamic eqns. (Rakavi et al. 1967)

$$\tau \;=\; \mathrm{quasi-time}, \qquad p = p(\rho, s, Y)$$

$$\frac{d}{dm}\frac{4\pi r^3}{3} = \frac{1}{\rho} \;\;, \qquad r_c = 0$$

$$\frac{\partial r}{\partial \tau} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{Gm}{r^2} \;\;, \qquad p_s = 0$$

For given distributions of entropy $s(m)$ and composition $Y_{1,\ldots J}(m)$

$$\int \left(\frac{\partial r}{\partial \tau}\right)^2 dm = -\frac{\partial}{\partial \tau}\int u\,dm + \frac{\partial}{\partial \tau}\int \frac{Gm\,dm}{r}$$

$$\Downarrow$$

Integration continually reduces

$$E = \int u\,dm - \int \frac{Gm\,dm}{r}$$

$$\Downarrow$$

Minimum $E \Longleftrightarrow$ Hydrostatic equilibrium

or

No minimum $\Longrightarrow$ Dynamical instability

'Non-canonical' models (cont.)

The quasi-dynamic method

- either finds hydrostatic equilibrium or else detects dynamical instability!
- works from any initial $r(m)$ distribution, e.g. one of arbitrary, uniform density
- is very efficient: iteration on implicit difference eqns leads to tridiagonal matrix
- is ideally suited for creation of initial models

Recipe

quasi-dynamic mill

$$\Downarrow$$

$$\text{hydrostatic} \quad \text{structure}: \quad (r_i, m_i, p_i, [Y_j]_i)$$

$$\Downarrow$$

$$T_i = T(\rho_i, s_i, [Y_j]_i)$$

$$\Downarrow$$

$$L_i = \int_0^{m_i} [q(p,T,Y) - q_\nu(p,T,Y)]\, dm$$

Outstanding problems

The stellar evolution code cannot yet deal with

- white dwarfs, when the outer mass shells become less than $10^{-15}\mathrm{M}_\odot$, because the interpolation required to determine $u^0(m_j)$ from $u^0(m_j^0)$ fails when the m_j cease to be monotonic. This is a minor technical nuisance.
- super-Eddington luminosities ($\Gamma > 1$, due to high κ or L), when the boundary condition

$$(\kappa p_G)_s = (1 - \Gamma_s)\tau_s g_s$$

  fails to deliver a positive pressure, as it must. In this case a so-called thick wind must begin to blow.

Are the huge jumps in Alexander and Ferguson's low temperature opacities—e.g. more than seven orders of magnitude for a change of $\Delta T = 100\,K$at $T = 1200\,K$—plausible?