

1. What are the masses of the Earth, the Sun and the Milky Way galaxy in GeV? (Since most of the mass of ordinary matter comes from the rest mass of their constituent protons and neutrons, and since these have masses close to a GeV, what you find would be the total number of nucleons in them if they were completely made of ordinary matter.)

The total energy density of the universe is measured in cosmology to be around 9×10^{-30} g/cm³. What is this in (meV)⁴?

2. Fill in the missing factors of Boltzmann's constant, k_B , Planck's constant, \hbar , and the speed of light, c , in the following formulae:
 - (a) $\lambda = (mT)^{-1/2}$ where λ is a length scale, m is a mass and T is a temperature.
 - (b) $E = G^2 Mm/r^3$ where E is an energy, r is a distance, M and m are masses and G is Newton's gravitational constant.

Based on what you find, do you expect either of these to be quantum effects?

3. For a one-dimensional simple harmonic oscillator, the ladder operator has the following explicit representation as a differential operator:

$$\mathcal{A} = \frac{1}{\sqrt{2m\omega}}(m\omega \mathcal{X} + i\mathcal{P}) = \frac{1}{\sqrt{2m\omega}} \left(m\omega x + \frac{\partial}{\partial x} \right).$$

- (a) Use this and its adjoint to compute the vacuum wave-function by regarding the condition $\mathcal{A}\psi_0(x) = 0$ as a differential equation and solving it. (Do not obtain ψ_0 by solving the Schrödinger equation.)
 - (b) Explicitly compute the first (and second) excited states, $\psi_1(x)$ and $\psi_2(x)$ from $\psi_0(x)$ by acting on it once (or twice) with the operator \mathcal{A}^\dagger . Do these agree with the standard wave-functions for the lowest three levels as found by solving the Schrödinger equation? (You need not solve the Schrödinger equation; it suffices to compare to a reference, though cite your reference in your solution.)
 - (c) Write out the operator $\mathcal{A}^\dagger \mathcal{A}$ explicitly as a differential operator. Apply this operator explicitly to your states $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ and prove these are eigenfunctions of $\mathcal{A}^\dagger \mathcal{A}$ (and, by doing so, evaluate their eigenvalues).
4. Consider a system of 3 indistinguishable particles (in 1 spatial dimension) that all interact with a potential $V(x) = \frac{1}{2} m\omega^2 x^2$ but do not interact with one another. That is, take the hamiltonian to be

$$\mathcal{H} = -\frac{1}{2m} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) + \frac{1}{2} m\omega^2 (x_1^2 + x_2^2 + x_3^2).$$

The 1-particle problem has energy eigenstates $\psi_n(x)$ with energy $E_n = (n + \frac{1}{2})\omega$.

- (a) Compute the lowest energy eigenvalue for the 3-particle system assuming the particles are bosons. What are the energies of the first and second 3-particle excited states (also for bosons)?

- (b) Write down explicitly the wavefunctions, $\psi(x_1, x_2, x_3)$ for *all* of the possible states with these energies (*i.e.* ground, first-excited and second-excited states), in terms of the single-particle wavefunctions $\psi_n(x)$. How many independent states are there for each of these three energies?
- (c) How many states would you have for the ground-state and first two excited states if there instead were N indistinguishable particles interacting with the harmonic oscillator potential

$$V(x_1, \dots, x_N) = \frac{1}{2} m\omega^2 (x_1^2 + \dots + x_N^2) ?$$

- (d) Repeat this question, but now assuming the indistinguishable particles are fermions.