1. (a) What are the masses of the Moon and the Sun in GeV? (This is roughly the total number of nucleons in them assuming they were completely made of ordinary matter.)

   (b) The magnetic field at the surface of the Sun can be around 0.3 T. How large is this in GeV^2?

   (c) The muon has a rest mass of $m = 106$ MeV and it has a mean decay lifetime of $\tau = 2.20 \times 10^{-6}$ sec. The dimensionless quantity built from $m\tau$ measure of the strength of the interaction that is responsible for muon decay. How large is $m\tau$ numerically?

   (d) How large is τ in metres? This is an average upper limit on the distance the muon would travel before decaying (because the muon always moves slower than the speed of light). How does the distance you find compare to the thickness of the Earth's atmosphere (which is roughly 100 km)?

   Muons are produced at the top of the atmosphere by reactions when protons from the solar wind collide with atoms in the air. Based on your answer above, should the produced muons usually reach the ground? (It turns out that they do. Can you think of a reason why they should?)

   (e) What is room temperature (273 K) in eV? Is this larger or smaller than the ionization energy for Hydrogen? Based on this comparison is it reasonable that air is not ionized at room temperature?

   The solar surface is at around 6000 K. What is this in eV? Do you expect Hydrogen to be ionized at the solar surface?

   (f) A calculation yields $\delta E = G^2 Mm/r^3$ where δE is an interaction energy, r is the distance between two bodies of mass M and m while G is Newton's gravitational constant. Fill in the missing factors of Boltzmarnn's constant, k_B, Planck's constant, $\hbar$, and the speed of light, c, in the formula for δE.

   Based on your answer, is δE a classical or quantum contribution to the interaction energy?

2. For a one-dimensional simple harmonic oscillator, the ladder operator has the following explicit representation as a differential operator:

$$\mathcal{A} = \frac{1}{\sqrt{2m\omega}}(m\omega\,\mathcal{X} + i\mathcal{P}) = \frac{1}{\sqrt{2m\omega}}\left(m\omega x + \frac{\partial}{\partial x}\right) .$$

   (a) Find the wavefunction $\psi_\alpha(x)$ for the eigenstates that satisfy $\mathcal{A}\psi_\alpha = \alpha\psi_\alpha$.

   (b) Write out the operator $\mathcal{A}^\star\mathcal{A}$ explicitly as a differential operator. Suppose a function $\psi(x) = H(x)\,\exp[-\frac{1}{2}\,m\omega x^2]$ satisfies $\mathcal{A}^\star\mathcal{A}\,\psi = 2\psi$. What differential equation must $H(x)$ satisfy? Solve this to find $H(x)$ assuming H is a polynomial in x.

   (c) Calculate $\phi(x)$ where $\phi = \mathcal{A}^\star\,\psi$. Show that ϕ is an eigenstate of $\mathcal{A}^\star\mathcal{A}$ and compute its eigenvalue.

3. Consider the following 1-dimensional cartoon of the tritium nucleus, in which a proton and two neutrons sit within a 1-dimensional harmonic oscillator potential, with hamiltonian

$$H = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right) + \frac{1}{2}\, m\omega^2\left(x_1^2 + x_2^2 + x_3^2\right)\,.$$

   Here x_1 is the position of the proton while x_2 and x_3 are the positions of the two neutrons. Inter-nucleon interactions are neglected and protons and neutrons are both fermions.

   Keeping in mind that multi-particle solutions $\Psi(x_1, x_2, x_3)$ to $H\Psi = E\Psi$ can be built by taking products of single particle solutions, $\psi_n(x_1)$, $\psi_m(x_2)$ and $\psi_r(x_3)$, what is the wave-function and energy eigenstate for the ground state of this three-particle system?

   What would have been the same answer if neutrons were bosons and not fermions?

4. Suppose a_p and $a_p^\star$ and b_p and $b_p^\star$ are respectively the annihilation and creation operators for two types of particles, and both are bosons. Suppose further that both types of particles have exactly the same mass, m, and exactly opposite electric charge $\pm e$. (Their masses and charges would be related to each other in this way if one was the antiparticle of the other, as it happens.)

   (a) Prove that the three-particle state $|\psi\rangle = b_q^\star a_p^\star b_q^\star |0\rangle$ is an eigenstate of the energy and charge operators

   $$H = \sum_k \sqrt{k^2 + m^2}\,\left(a_k^\star a_k + b_k^\star b_k\right) \quad \text{and} \quad Q = e\sum_k \left(a_k^\star a_k - b_k^\star b_k\right). \tag{0.1}$$

   What are their eigenvalues for the state $|\psi\rangle$?

   (b) Compute the vacuum expectation value $\langle 0|\mathcal{O}|0\rangle$ of the operator

   $$\mathcal{O} = \sum_{pq} A_{pq}(u_p a_p + v_p b_p^\star)^\star (u_q a_q + v_q b_q^\star)$$

   where $A_{pq} = A_{qp}^*$, u_p and v_p can all be regarded as known functions of p and q.

   (c) Compute the expectation value $\langle\phi|\mathcal{O}|\phi\rangle$ of the same operator but now in the state

   $$|\phi\rangle = |0\rangle + \sum_{kl} C_{kl} a_k^\star b_l^\star |0\rangle\,.$$

   Here C_{pq} is another known function of p, q.