1. Consider the following hamiltonian describing the decay process $B \to A + A$ where A and B are two particle types: $H = H_{\text{free}} + H_{\text{int}}$ where

$$H_{\text{free}} = E_0 + \int \mathrm{d}^3p \Big[\varepsilon_B(p)\, b_p^\star b_p + \varepsilon_A(p)\, a_p^\star a_p\Big] ,$$

and

$$H_{\text{int}} = g \int \frac{\mathrm{d}^3p\, \mathrm{d}^3q\, \mathrm{d}^3k}{\sqrt{\varepsilon_B(p)\varepsilon_A(q)\varepsilon_A(k)}} \Big[b_p\, a_q^\star\, a_k^\star + a_k\, a_q\, b_p^\star\Big] \delta^3(\mathbf{p} - \mathbf{q} - \mathbf{k}) ,$$

for real constants E_0 and g and dispersion relations

$$\varepsilon_B(p) = \sqrt{\mathbf{p}^2 + m^2} \quad \text{and} \quad \varepsilon_A(p) = |\mathbf{p}| .$$

Treat both types of particle as bosons.

   (a) For a decay with momentum assignments $B(\mathbf{p} = 0) \to A(\mathbf{q}) + A(\mathbf{k})$ use Fermi's Golden Rule to compute the differential decay $\mathrm{d}\Gamma/\mathrm{d}^3k$ as a function of one of the final-particle momenta.

   (b) Perform the integration over final-state momenta to compute the total rate, Γ, for the decay. (For indistinguishable final-state particles is it correct to integrate the final momentum over the usual 4π solid angle?)

   (c) If $m = 150$ MeV and $g = 0.1$ what is the expected mean lifetime, $\tau = 1/\Gamma$, for this decay in seconds?

2. Suppose particles interact with a time-dependent oscillating potential, $V(t) = V_0 \cos(\Omega t)$, where V_0 and Ω are positive real numbers, is described by the Schrödinger-picture hamiltonian

$$H = E_0 + \int \mathrm{d}^3p \Big[\varepsilon(p)\, a_p^\star a_p + V(t)(a_p a_{-p} + a_p^\star a_{-p}^\star)\Big] .$$

Assume the relativistic dispersion relation

$$\varepsilon(p) = \sqrt{\mathbf{p}^2 + m^2} ,$$

and treat the particles as fermions. Compute the rate for the pair-production process in which the no-particle state $|0\rangle$ evolves into a state $|k, -k\rangle = a_k^\star a_{-k}^\star |0\rangle$, treating $V(t)$ in time-dependent perturbation theory. What is the minimum value of Ω for which pair production occurs? Notice that because H is already time-dependent in Schrödinger picture you cannot directly use Fermi's Golden rule (since the time-dependence of H implies particle energy is not conserved in this process). So be sure to start your derivation from first principles.