1. a)

(*Speed of light in m/s*)

In[ ]:= c = 299 792 458 ;

(* $\frac{\text{Gev}}{c^2}$ = x kg*)

(* x = $\frac{\text{Gev}}{c^2}\frac{1}{\text{kg}}$*)

In[ ]:= $x = \frac{1.602176634 * 10^{-19} * 10^{9}}{c^2}$;

(*Mass of moon and sun in kg*)

In[ ]:= Mmoon = $7.342 * 10^{22}$;
Msun = $1.9885 * 10^{30}$;

(*Mass of moon in GeV*)

In[ ]:= $\frac{\text{Mmoon}}{x}$

Out[ ]= 4.11856×10^{49}

(*Mass of sun in GeV*)

In[ ]:= $\frac{\text{Msun}}{x}$

Out[ ]= 1.11547×10^{57}

1.b)

(* kg = $\frac{1}{x}$ $\frac{\text{Gev}}{c^2}$*)

(*Reduced Planck's constant in eV*s*)

(* $\hbar$= y eV*s *)

y = $6.5821 * 10^{-16}$;

(* s = $\frac{1}{y}\frac{\hbar}{\text{eV}}$= $\frac{1}{y}$ eV^{-1} =$\frac{1}{y}$ GeV^{-1} 10^{9}*)

(* C= $6.24151*10^{18}$ e = $6.24151*10^{18}$*)

(*T=$\frac{N*s}{C*m}$=$\frac{kg}{C*s}$=$\frac{y}{x}$ $\frac{10^{-9}}{6.24151*10^{18}}$ GeV^2*)

In[ ]:= $\frac{y}{x}\frac{10^{-9}}{6.24151*10^{18}}$

Out[ ]= 5.9157×10^{-17}

(*Magnetic field at surface of Sun in T*)

In[ ]:= BSun = 0.3;

(*Magnetic field at surface of Sun in GeV^2*)

In[ ]:= BSun $*\frac{y}{x}\frac{10^{-9}}{6.24151*10^{18}}$

Out[ ]= 1.77471×10^{-17}

1.c)

(*Mmuon in eV*)

In[ ]:= Mmuon = $106*10^{6}$;

(*Reduced Planck's constant in eV*s*)

(* ħ= y eV*s *)

In[ ]:= y = $6.5821*10^{-16}$;

(*τ in s*)

In[ ]:= τ = $2.2*10^{-6}$;

(* s = $\frac{1}{y}\frac{\hbar}{eV}$= $\frac{1}{y}$ eV^{-1} *)

(*τ in eV^{-1}*)

In[ ]:= $\frac{\tau}{y}$

Out[ ]= 3.3424×10^{9}

In[ ]:= (* Mmuon*τ *)

In[ ]:= Mmuon $*\frac{\tau}{y}$

Out[ ]= 3.54294×10^{17}

1. d)

(*c=z $\frac{m}{s}$*)

In[ ]:= z = c;

(*s = z $\frac{m}{c}$ =z m*)

(*τ in metres*)

In[ ]:= τ * z

Out[ ]= 659.543

(* ~660 m < 100 km, based on this muons shouldn't reach ground*)

1.e)

In[ ]:= (* $\frac{eV}{kB}$= u K*)

(* K= $\frac{1}{u}$ $\frac{eV}{kB}$= $\frac{1}{u}$ eV*)

$$u = \frac{1.602176634 * 10^{-19}}{1.380649 * 10^{-23}}$$

Out[ ]= 11 604.5

In[ ]:= (*Room temperature in K*)

Troom = 273;

In[ ]:= (*Room temperature in eV*)

$$\frac{Troom}{u}$$

Out[ ]= 0.0235253

(* ~0.024 eV < 13.6 eV, it's reasonable that air is not ionized*)

(*Temperature of the surface of the Sun in K*)

In[ ]:= TSun = 6000;

(*Temperature of the surface of the Sun in eV*)

In[ ]:= $$\frac{TSun}{u}$$

Out[ ]= 0.51704

(*~0.52 eV < 13.6 eV, Hydrogen isn't ionized at the solar surface*)

1.f)

(* $[\delta E]=J=kg*\frac{m^2}{s^2}$*)

(* $[G]= \frac{N*m^2}{kg^2}=\frac{m^3}{kg\ s^2}$*)

(*[M]=[m]=kg*)

(*[r]=m*)

(* $[\hbar]=J*s =kg*\frac{m^2}{s}$ *)

(* $[c]=\frac{m}{s}$*)

(* $[kB]= \frac{J}{K}=\frac{kg*m^2}{s^2 K}$*)

(* $kg*\frac{m^2}{s^2} = [kB]^\alpha[c]^\beta[\hbar]^\gamma \frac{m^6}{kg^2\ s^4} kg^2 \frac{1}{m^3} = [kB]^\alpha[c]^\beta[\hbar]^\gamma \frac{m^3}{s^4}$*)

(* $[kB]^\alpha[c]^\beta[\hbar]^\gamma=kg*\frac{m^2}{s^2}\frac{s^4}{m^3}=kg*\frac{s^2}{m}$*)

(* $\alpha=0$, $\beta=-3$, $\gamma= 1$*)

(* $\delta E=\frac{G^2 M\ m}{r^3}\frac{\hbar}{c^3}$, this is a quantum contribution to the energy because of $\hbar$*)

2.

$$\hat{A} = \frac{1}{\sqrt{2m\omega}} (m\omega \, \hat{X} + i \, \hat{p})$$

$$= \frac{1}{\sqrt{2m\omega}} \left(m\omega x + \frac{\partial}{\partial x} \right)$$

(a)

$$A \psi_\alpha = \frac{1}{\sqrt{2m\omega}} \left(m\omega x + \frac{\partial}{\partial x} \right) \psi_\alpha(x) = \alpha \psi_\alpha(x)$$

$$\frac{d\psi_\alpha}{dx} = (\alpha \sqrt{2m\omega} - m\omega x) \psi_\alpha(x)$$

$$\frac{d\psi_\alpha}{\psi_\alpha} = (\alpha \sqrt{2m\omega} - m\omega x) \, dx$$

normalization factor

$$\boxed{\psi_\alpha(x) = N \, e^{\alpha x \sqrt{2m\omega} - \frac{1}{2} m\omega x^2}}$$

(b)

$$A^* A = \frac{1}{2m\omega} \left(m\omega x - \frac{\partial}{\partial x} \right) \left(m\omega x + \frac{\partial}{\partial x} \right)$$

$$= \frac{1}{2m\omega} \left((m\omega x)^2 - m\omega - \cancel{m\omega x \frac{\partial}{\partial x}} + \cancel{m\omega x \frac{\partial}{\partial x}} - \frac{\partial^2}{\partial x^2} \right)$$

(b) $A^*A = \frac{1}{2m\omega}\left(m\omega x - \frac{\partial}{\partial x}\right)\left(m\omega x + \frac{\partial}{\partial x}\right)$

$$= \frac{1}{2m\omega}\Big((m\omega x)^2 - m\omega - \cancel{m\omega x \frac{\partial}{\partial x}} + \cancel{m\omega x \frac{\partial}{\partial x}} - \frac{\partial^2}{\partial x^2} \Big)$$

$\psi(x) = H(x)\, e^{-\frac{1}{2}m\omega x^2}$

$(A^*A - 2)\psi = 0$:

$$\frac{1}{2m\omega}\left((m\omega x)^2 - 5m\omega - \frac{\partial^2}{\partial x^2}\right) H(x)\, e^{-\frac{1}{2}m\omega x^2} = 0$$

$$\left((m\omega x)^2 - 5m\omega\right) H(x)\, e^{-\frac{1}{2}m\omega x^2}$$

$$- \frac{\partial}{\partial x}\left(H'(x)\, e^{-\frac{1}{2}m\omega x^2} - m\omega x\, H(x)\, e^{-\frac{1}{2}m\omega x^2}\right)$$

$$= \left(\cancel{(m\omega x)^2} - \overset{4}{\cancel{5}}\, m\omega\right) H(x)\, e^{-\frac{1}{2}m\omega x^2}$$

$$- \Big(H''(x)\, e^{-\frac{1}{2}m\omega x^2} - 2m\omega x\, H'(x)\, e^{-\frac{1}{2}m\omega x^2} - \cancel{m\omega H(x)\, e^{-\frac{1}{2}m\omega x^2}} + \cancel{(m\omega x)^2 H(x)\, e^{-\frac{1}{2}m\omega x^2}}\Big) =$$

$$= -\left(H''(x) - 2m\omega x\, H'(x) + 4m\omega H\right) e^{-\frac{1}{2}m\omega x^2} = 0$$

$$\boxed{H''(x) - 2m\omega x\, H'(x) + 4m\omega H(x) = 0}$$

$$H''(x) - 2m\omega x\, H'(x) + 4m\omega\, H(x) = 0$$

- Ansatz:

$$H(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=0}^{\infty} \left(n(n-1)\, a_n\, x^{n-2} - 2m\omega\, n\, a_n x^n + 4m\omega\, x^n a_n \right)$$

$$= \sum_{n=0}^{\infty} \left((n+2)(n+1)\, a_{n+2} - 2m\omega\, (n-2)\, a_n \right) x^n = 0$$

$$\Rightarrow \quad a_{n+2} = \frac{2m\omega\,(n-2)}{(n+2)(n+1)}\, a_n$$

$$a_2 = -\frac{4m\omega}{2}\, a_0 = -2m\omega\, a_0,$$

$a_4 = 0,$

$a_{2m+2} = 0 \quad , \quad m \in \mathbb{N} \, , \; m > 0$

$$a_3 = -\frac{2m\omega}{3\cdot 2}\, a_1 = -\frac{m\omega}{3}\, a_1$$

$a_{2m+1+2} \propto a_1 \, , \quad m \in \mathbb{N}$

↳ if we demand that ψ is normalizable, $a_1 = 0$

arbitrary phase

$$\Rightarrow H(x) = a_0 \left(1 - 2m\omega x^2\right)$$
$$= e^{i\alpha} \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi}\right)^{1/4} \left(1 - 2m\omega x^2\right)$$

follows when we normalize ψ

(c) $\phi \equiv A^* \psi$

$$A^* A \phi = A^* A (A^* \psi)$$

$$= A^* (1 + A^* A) \psi$$

$$= A^* \psi + A^* \overbrace{(A^* A) \psi}^{= 2\psi}$$

$$= 3 A^* \psi$$

$$A^* \psi = \frac{m\omega x - \frac{\partial}{\partial x}}{\sqrt{2m\omega}} \left(\frac{m\omega}{\pi}\right)^{\frac{1}{4}} \frac{e^{i\phi}}{\sqrt{2}} (1 - 2m\omega x^2)\, e^{-\frac{1}{2}m\omega x^2}$$

$$= \left(\frac{m\omega}{\pi}\right)^{\frac{1}{4}} \sqrt{m\omega}\, x \,(3 - 2m\omega x^2)\, e^{-\frac{1}{2}m\omega x^2} e^{i\phi}$$

3.

$$H = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right) + \frac{1}{2} m\omega \left(x_1^2 + x_2^2 + x_3^2\right)$$

The wavefunction for the ground state is:

$$\Psi(x_1, x_2, x_3) = \psi_0(x_1) \frac{1}{\sqrt{2}} \left(\psi_0(x_2)\psi_1(x_3) - \psi_1(x_2)\psi_0(x_3)\right)$$

neutrons can't be in the same state since they are identical fermions

and

$$H\Psi(x_1, x_2, x_3) = \frac{5\omega}{2}\Psi(x_1, x_2, x_3)$$

sum of single particle energies, since system is non-interacting

If the neutrons were bosons, the wavefunction would be

$$\Psi'(x_1, x_2, x_3) = \psi_0(x_1)\psi_0(x_2)\psi_0(x_3)$$

and

$$H\Psi'(x_1, x_2, x_3) = \frac{3\omega}{2}\Psi'(x_1, x_2, x_3)$$

4.

(a)

$$H|\psi\rangle$$

$$= \sum_k \sqrt{k^2+m^2}\,(a_k^* a_k + b_k^* b_k)\, b_q^* a_p^* b_q^* |0\rangle$$

$$= \sum_k \sqrt{k^2+m^2}\,\Big(a_k^* \underbrace{a_k a_p^*}_{=\,\delta_{k,p} + a_p^* a_k} b_q^* b_q^* + b_k^* \underbrace{b_k b_q^*}_{=\,\delta_{k,q} + b_q^* b_k} a_p^* b_q^*\Big)|0\rangle$$

$$= \sqrt{p^2+m^2}\; \overbrace{a_p^* b_q^* b_q^* |0\rangle}^{=\, b_q^* a_p^* b_q^*} + \sqrt{q^2+m^2}\; b_q^* a_p^* b_q^* |0\rangle$$

$$+ \sum_k \sqrt{k^2+m^2}\,\Big(a_k^* a_p^* \overbrace{a_k b_q^* b_q^* |0\rangle}^{=\, b_q^* b_q^* a_k |0\rangle \,=\, 0} + b_k^* b_q^* \underbrace{b_k a_p^* b_q^*}_{=\, a_p^* \underbrace{b_k b_q^*}_{=\,\delta_{k,q} + b_q^* b_k}} |0\rangle \Big)$$

$$= \left(\sqrt{p^2+m^2} + \sqrt{q^2+m^2}\right) b_q^* a_p^* b_q^* |0\rangle$$

$$+ \sum_k \sqrt{k^2+m^2}\,\Big(\delta_{k,q}\; \overbrace{b_k^* b_q^* a_p^* |0\rangle}^{=\, b_k^* a_p^* b_q^* |0\rangle} + b_k^* b_q^* a_p^* b_q^* \overbrace{b_k |0\rangle}^{=\,0} \Big)$$

$$= \left(2\sqrt{q^2+m^2} + \sqrt{p^2+m^2}\right) b_q^* a_p^* b_q^* |0\rangle$$

$$= \left(2\sqrt{q^2+m^2} + \sqrt{p^2+m^2}\right) |\psi\rangle$$

$a|\psi\rangle$

$$= e \sum_k \left(a_k^* a_k - b_k^* b_k \right) b_q^* a_p^* b_q^* |0\rangle$$

$$= e \sum_k \left(a_k^* \underbrace{a_k a_p^*}_{= \delta_{k,p} + a_p^* a_k} b_q^* b_q^* - b_k^* \underbrace{b_k b_q^*}_{= \delta_{k,q} + b_q^* b_k} a_p^* b_q^* \right) |0\rangle$$

$$= e \cancel{\overbrace{a_p^* b_q^* b_q^*}^{= b_q^* a_p^* b_q^*}} |0\rangle - e \cancel{b_q^* a_p^* b_q^*} |0\rangle$$

$$+ e \sum_k \Big(a_k^* a_p^* \overbrace{a_k b_q^* b_q^* |0\rangle}^{= b_q^* b_q^* a_k |0\rangle = 0}$$

$$- b_k^* b_q^* \underbrace{b_k a_p^* b_q^*}_{= a_p^* \underbrace{b_k b_q^*}_{= \delta_{k,q} + b_q^* b_k}} |0\rangle \Big)$$

$$= -e \sum_k \Big(\delta_{k,q} \overbrace{b_k^* b_q^* a_p^* |0\rangle}^{= b_k^* a_p^* b_q^* |0\rangle}$$

$$+ \cancel{b_k^* b_q^* a_p^* \overbrace{b_q^* b_k |0\rangle}^{= 0}} \Big)$$

$$= -e \, b_q^* a_p^* b_q^* |0\rangle$$

$$= -e |\psi\rangle$$

(b)

$$O = \sum_{p,q} A_{pq} (u_p a_p + v_p b_p^*)^* (u_q a_q + v_q b_q^*)$$

$$= \sum_{p,q} A_{pq} (u_p^* u_q \; a_p^* a_q + u_p^* v_q \; a_p^* b_q^*$$

$$+ v_p^* u_q \; b_p a_q + v_p^* v_q \; b_p b_q^*)$$

$$\langle 0 | O | 0 \rangle$$

$$= \sum_{p,q} \langle 0 | A_{pq} (u_p^* u_q \; \cancel{a_p^* a_q} + u_p^* v_q \; \cancel{a_p^* b_q^*}$$

$$+ v_p^* u_q \; \cancel{b_p a_q} + v_p^* v_q \; \underbrace{b_p b_q^*}_{= \delta_{p,q} + b_q^* b_p}) | 0 \rangle$$

$$= \sum_p A_{pp} \, v_p^* v_p \; \overbrace{\langle 0 | 0 \rangle}^{= 1}$$

$$+ \sum_{p,q} A_{pq} \, v_p^* v_q \; \overbrace{\langle 0 | b_q^* b_p | 0 \rangle}^{= 0}$$

$$= \sum_p A_{pp} \, v_p^* v_p$$

(c)

$$|\phi\rangle = |0\rangle + \sum_{k,l} c_{kl}\, a_k^* b_l^* |0\rangle$$

$$\begin{aligned}
\langle \phi | O | \phi \rangle = {} & \langle 0|O|0\rangle + \sum_{k,l} c_{kl} \langle 0|O\, a_k^* b_l^* |0\rangle \\
& + \sum_{k,l} c^*_{kl} \langle 0| b_l a_k\, O |0\rangle \\
& + \sum_{k,l} \sum_{k',l'} \langle 0| b_l a_k\, O\, a_{k'}^* b_{l'}^* |0\rangle \cdot c^*_{kl} c_{k'l'} \\
= {} & \sum_p A_{pp} v_p^* v_p + \sum_{k,l} c_{kl} A_{lk} v_l^* u_k \\
& + \sum_{k,l} c^*_{kl} A_{kl} u_k^* v_l \\
& + \sum_{k,l} \sum_{k',l'} c^*_{kl} c_{k'l'} \Big(A_{kk'} u_k^* u_{k'} \delta_{l,l'} \\
& + A_{l'l} v_{l'}^* v_l \delta_{k,k'} + \sum_p A_{pp} v_p^* v_p \delta_{l,l'} \delta_{k,k'} \Big) \\
= {} & \sum_p A_{pp} v_p^* v_p \Big(1 + \sum_{k,l} |c_{kl}|^2 \Big) \\
& + \sum_{k,l} \left(c_{kl} A_{lk} v_l^* u_k + c^*_{kl} A_{kl} u_k^* v_l \right) \\
& + \sum_l \sum_{k,k'} \Big(c^*_{kl} c_{k'l} A_{kk'} u_k^* u_{k'} \\
& \qquad + c^*_{lk} c_{lk'} A_{k'k} v_{k'}^* v_k \Big)
\end{aligned}$$

shown on next few pages

$$\langle 0 | O \, a_k^* b_l^* | 0 \rangle$$

$$= \sum_{p,q} \langle 0 | A_{pq} \left(u_p^* u_q \, a_p^* a_q + u_p^* v_q \, a_p^* b_q^* + v_p^* u_q \, b_p a_q + v_p^* v_q \, b_p b_q^* \right) a_k^* b_l^* | 0 \rangle$$

$$= \sum_{p,q} A_{pq} \langle 0 | v_p^* u_q \, \underbrace{a_q a_k^*}_{\delta_{q,k} + a_k^* a_q} \, \underbrace{b_p b_l^*}_{= \delta_{p,l} + b_l^* b_p} | 0 \rangle$$

$$= \sum_q A_{lq} \langle 0 | v_l^* u_q \, a_q a_k^* | 0 \rangle + \sum_{pq} A_{pq} \cancel{\langle 0 | v_p^* u_q \, a_q a_k^* \underbrace{b_l^* b_p | 0 \rangle}_{=0}}$$

$$= A_{lk} v_l^* u_k + \sum_q A_{lq} \cancel{\langle 0 | v_l^* v_q \, \underbrace{a_k^* a_q | 0 \rangle}_{=0}}$$

where we use:

$$\underbrace{\langle 0 | a_p^*}_{=0} a_q a_k^* b_l^* | 0 \rangle = 0$$

$$\underbrace{\langle 0 | a_p^*}_{=0} b_q^* a_k^* b_l^* | 0 \rangle = 0$$

$$\langle 0 | b_p b_q^* a_k^* b_l^* | 0 \rangle$$

$$= \underbrace{\langle 0 | a_k^*}_{=0} b_p b_q^* b_l^* | 0 \rangle = 0$$

$$\langle 0 | b_l a_k \, O \, | 0 \rangle$$

$$= \sum_{pq} A_{pq} \langle 0 | b_l a_k \big(u_p^* u_q \cancel{a_p^* a_q} + u_p^* v_q a_p^* b_q^* + v_p^* u_q \cancel{b_p a_q} + v_p^* v_q b_p b_q^* \big) | 0 \rangle$$

$$= \sum_{p,q} A_{pq} u_p^* v_q \langle 0 | a_k a_p^* \underbrace{b_l b_q^* | 0 \rangle}_{= (\delta_{l,q} + \cancel{b_q^* b_l}) | 0 \rangle}$$

$$= \sum_{p} A_{pl} u_p^* v_l \langle 0 | \underbrace{a_k a_p^* | 0 \rangle}_{(\delta_{k,p} + \cancel{a_p^* a_k}) | 0 \rangle}$$

$$= A_{kl} u_k^* v_l$$

where we use:

$$\langle 0 | b_l a_k a_p^* \overbrace{a_q | 0 \rangle}^{= 0} = 0$$

$$\langle 0 | b_l a_k b_p \overbrace{a_q | 0 \rangle}^{= 0} = 0$$

$$\langle 0 | b_l a_k b_p b_q^* | 0 \rangle$$

$$= \langle 0 | b_l b_p b_q^* \overbrace{a_k | 0 \rangle}^{= 0} = 0$$

$$\langle 0 | b_l a_k \, O \, a_{k'}^* b_{l'}^* | 0 \rangle$$

$$= \sum_{p,q} A_{pq} \langle 0 | b_l a_k \big(u_p^* u_q \, a_p^* a_q + u_p^* v_q \, a_p^* b_q^* + v_p^* u_q \, b_p a_q + v_p^* v_q \, b_p b_q^* \big) a_{k'}^* b_{l'}^* | 0 \rangle$$

$$= \sum_{p,q} A_{pq} \big(u_p^* u_q \langle 0 | b_l a_k a_p^* a_q a_{k'}^* b_{l'}^* | 0 \rangle + v_p^* v_q \langle 0 | b_l a_k b_p b_q^* a_{k'}^* b_{l'}^* | 0 \rangle \big)$$

$$= \sum_{p,q} A_{pq} \big(u_p^* u_q \, \delta_{p,k} \delta_{q,k'} \delta_{l,l'} + v_p^* v_q \, \delta_{k,k'} (\delta_{q,l} \delta_{p,l'} + \delta_{l,l'} \delta_{p,q}) \big)$$

$$= A_{kk'} u_k^* u_{k'} \delta_{l,l'} + A_{l'l} v_{l'}^* v_l \delta_{k,k'} + \sum_p A_{pp} v_p^* v_p \, \delta_{l,l'} \delta_{k,k'}$$

where we use:

$$\langle 0 | b_l a_k a_p^* a_q a_{k'}^* b_{l'}^* | 0 \rangle$$

$$= \langle 0 | b_l a_k a_p^* b_{l'}^* | 0 \rangle \cdot \delta_{q,k'} + \langle 0 | b_l a_k a_p^* a_{k'}^* \underbrace{a_q b_{l'}^* | 0 \rangle}_{= b_{l'}^* a_q | 0 \rangle = 0}$$

$$= \langle 0 | a_k a_p^* \underbrace{b_l b_{l'}^* | 0 \rangle}_{= (\delta_{l,l'} + \cancel{b_{l'}^* b_l}) | 0 \rangle} \cdot \delta_{q,k'}$$

$$= \langle 0 | \underbrace{a_k a_p^* | 0 \rangle}_{= (\delta_{p,k} + \cancel{a_p^* a_k}) | 0 \rangle} \cdot \delta_{q,k'} \delta_{l,l'}$$

$$= \delta_{p,k} \delta_{q,k'} \delta_{l,l'}$$

$$\langle 0|\, b_l\, a_k\, b_p\, b_q^\dagger\, a_{k'}^\dagger\, b_{l'}^\dagger\, |0\rangle$$

$$= \langle 0|\, a_k\, a_{k'}^\dagger\, b_l\, b_p\, b_q^\dagger\, b_{l'}^\dagger\, |0\rangle$$

$$= \delta_{k,k'}\, \langle 0|\, b_p\, b_l\, b_q^\dagger\, b_{l'}^\dagger\, |0\rangle$$

$$= \delta_{k,k'}\,\delta_{q,l}\, \langle 0|\, b_p\, b_{l'}^\dagger\, |0\rangle$$

$$+ \delta_{k,k'}\, \langle 0|\, b_p\, b_q^\dagger\, b_l\, b_{l'}^\dagger\, |0\rangle$$

$$= \delta_{k,k'}\,\delta_{q,l}\,\delta_{p,l'} + \delta_{k,k'}\,\delta_{l',l}\, \langle 0|\, b_p\, b_q^\dagger\, |0\rangle$$

$$= \delta_{k,k'}\,(\delta_{q,l}\,\delta_{p,l'} + \delta_{l,l'}\,\delta_{p,q})$$

$$\langle 0|\, b_l\, a_k\, a_p^\dagger\, b_q^\dagger\, a_{k'}^\dagger\, b_{l'}^\dagger\, |0\rangle$$

$$= \delta_{k,p}\, \underbrace{\langle 0|\, b_l\, b_q^\dagger\, a_{k'}^\dagger\, b_{l'}^\dagger\, |0\rangle}_{0 \;=\; \langle 0|\, a_{k'}^\dagger\, b_l\, b_q^\dagger\, b_{l'}^\dagger\, |0\rangle} + \overbrace{\langle 0|\, b_l\, a_p^\dagger\, a_k\, b_q^\dagger\, a_{k'}^\dagger\, b_{l'}^\dagger\, |0\rangle}^{\langle 0|\, a_p^\dagger\, b_l\, a_k\, b_q^\dagger\, a_{k'}^\dagger\, b_{l'}^\dagger\, |0\rangle \;=}$$

$$= 0$$

$$\langle 0|\, b_l\, a_k\, b_p\, a_q\, a_{k'}^\dagger\, b_{l'}^\dagger\, |0\rangle$$

$$= \delta_{q,k'}\, \langle 0|\, b_l\, b_p\, b_{l'}^\dagger\, a_k\, |0\rangle + \langle 0|\, b_l\, a_k\, b_p\, a_{k'}^\dagger\, b_{l'}^\dagger\, a_q\, |0\rangle$$

$$= 0$$