

NB: Please note that it is your responsibility to know what the equations below mean and when they can be applied. Also note that not all equations provided below will be necessarily required for the questions on this test.

$$-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x} = 0$$

$$t' = \gamma(t - vx/c^2) \quad x' = \gamma(x - vt) \quad \text{where } \gamma = (1 - v^2/c^2)^{-1/2}$$

$$V^{x'} = \frac{V^x - v}{1 - vV^x/c^2} \quad V^{y'} = \frac{V^y}{1 - vV^x/c^2} \sqrt{1 - v^2/c^2} \quad V^{z'} = \frac{V^z}{1 - vV^x/c^2} \sqrt{1 - v^2/c^2}$$

$$ds^2 = g_{\alpha\beta}(x) dx^\alpha dx^\beta$$

$$\Gamma_{\beta\gamma}^\alpha = \frac{1}{2} g^{\alpha\delta} \left( \frac{\partial g_{\delta\beta}}{\partial x^\gamma} + \frac{\partial g_{\delta\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\delta} \right)$$

$$M(\text{in cm}) = \frac{G}{c^2} M(\text{in g})$$

$$\omega_\infty = \omega_* \left(1 - \frac{2M}{R}\right)^{\frac{1}{2}}$$

$$r_{ISCO} = 6M$$

$$\Omega^2 = M/r^3$$

$$\delta\phi_{prec} = \frac{6\pi G}{c^2} \frac{M}{a(1-\epsilon^2)}$$

$$\delta\phi_{def} = \frac{4GM}{c^2 b}$$

$$t = v - r - 2M \log \left| \frac{r}{2M} - 1 \right|$$

$$U = \left(\frac{r}{2M} - 1\right)^{1/2} e^{r/4M} \cosh(t/4M) \text{ and } V = \left(\frac{r}{2M} - 1\right)^{1/2} e^{r/4M} \sinh(t/4M) \text{ for } r > 2M$$

$$\frac{ds^\alpha}{d\tau} + \Gamma_{\beta\gamma}^\alpha s^\beta u^\gamma = 0$$

$$ds^2 = ds_{Schwarz}^2 - \frac{4GJ}{c^3 r^2} \sin^2 \theta (rd\phi)(cdt) + \dots$$

$$r_e(\theta) = M + \sqrt{M^2 - a^2 \cos^2 \theta}$$

$$\frac{df}{d\sigma} = \frac{dx^\alpha}{d\sigma} \frac{\partial f}{\partial x^\alpha}$$

$$a'^\beta = \frac{\partial x'^\beta}{\partial x^\alpha} a^\alpha, \quad a^\beta = \frac{\partial x^\beta}{\partial x'^\alpha} a'^\alpha$$

$$t'^\alpha{}_\beta = \frac{\partial x'^\alpha}{\partial x^\gamma} \frac{\partial x^\delta}{\partial x'^\beta} t^\gamma{}_\delta$$

$$\nabla_\alpha v^\beta = \frac{\partial v^\beta}{\partial x^\alpha} + \Gamma_{\alpha\gamma}^\beta v^\gamma, \quad \nabla_\alpha v_\beta = \frac{\partial v_\beta}{\partial x^\alpha} + \Gamma_{\alpha\beta}^\gamma v_\gamma$$

$$R_{\beta\gamma\delta}^\alpha = \frac{\partial \Gamma_{\beta\delta}^\alpha}{\partial x^\gamma} - \frac{\partial \Gamma_{\beta\gamma}^\alpha}{\partial x^\delta} + \Gamma_{\gamma\epsilon}^\alpha \Gamma_{\beta\delta}^\epsilon - \Gamma_{\delta\epsilon}^\alpha \Gamma_{\beta\gamma}^\epsilon$$

$$R_{\alpha\beta} \equiv R_{\alpha\gamma\beta}^\gamma$$

$$R \equiv R_\gamma^\gamma$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta}$$