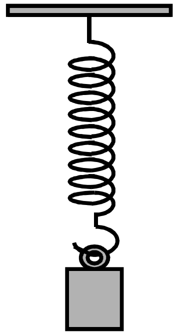
Damped Oscillations and Resonance

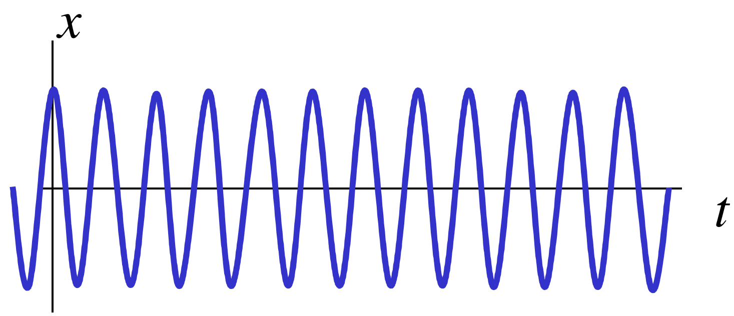
Serway 15.6, 15.7

- Damped Harmonic Oscillation
- Forced Oscillations
- Resonance

Practice: Chapter 15,
Conceptual Questions 7, 9
Problems 47, 53

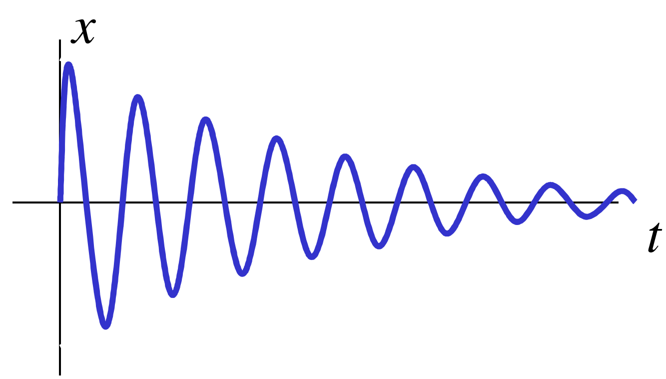
SHM: $x(t) = A \cos \omega t$



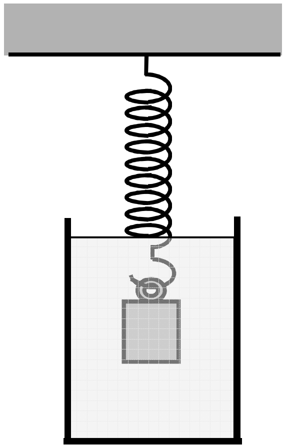


Motion continues indefinitely. Only conservative forces act, so the mechanical energy is constant.

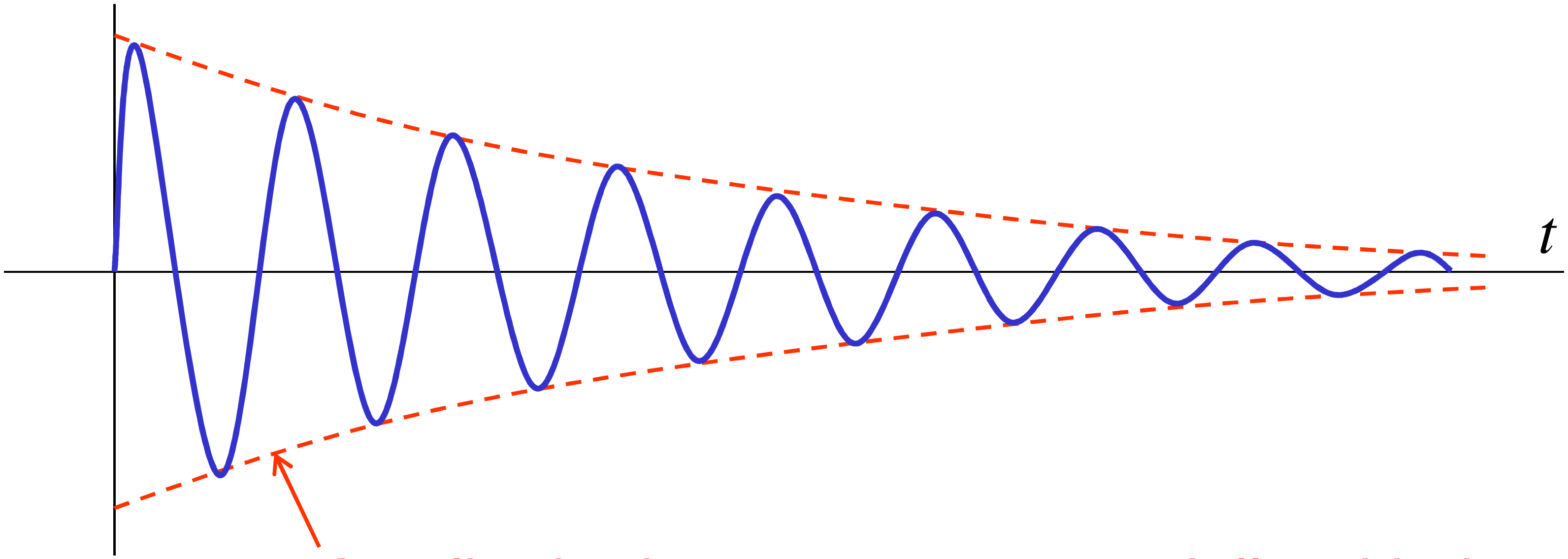
Damped oscillator: dissipative forces (friction, air resistance, *etc.*) remove energy from the oscillator, and the amplitude decreases with time.



Damped Oscillator: Drag force $\mathbf{f} = -b\mathbf{v}$



Weak damping: $x(t) = (A_0 e^{-\left(\frac{b}{2m}\right)t})\cos(\omega t + \phi)$



Without damping: the angular frequency is $\omega_0 = \sqrt{\frac{k}{m}}$

With damping: $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_0{}^2 - \left(\frac{b}{2m}\right)^2}$

The frequency ω is *slightly* lower with damping.

Example: A mass on a spring oscillates with initial amplitude 20 cm. After 10 seconds, the amplitude is 10 cm.

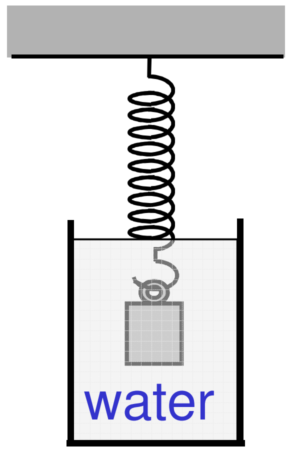
Question: What is the amplitude after 30 seconds?

A) 6.7 cm
B) 5 cm
C) 3.3 cm
D) 2.5 cm
E) zero

Question: What is the value of $b/(2m)$?

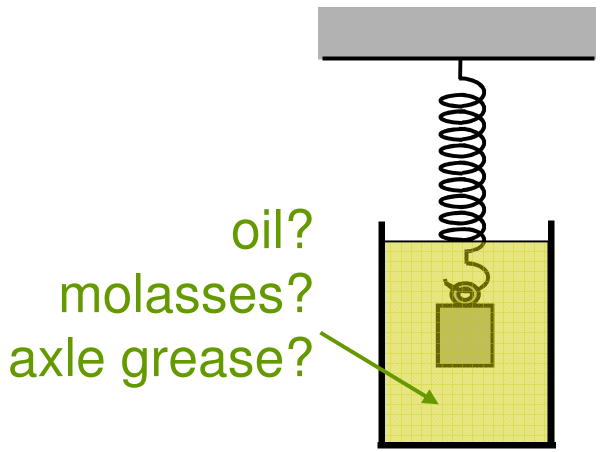
Weak damping: $x(t) = A_0 e^{-\frac{b}{2m}t} \cos(\omega t + \phi)$

with $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$



Strong damping (b large): there is **no oscillation** when

$$\frac{b}{2m} \geq \omega_0$$

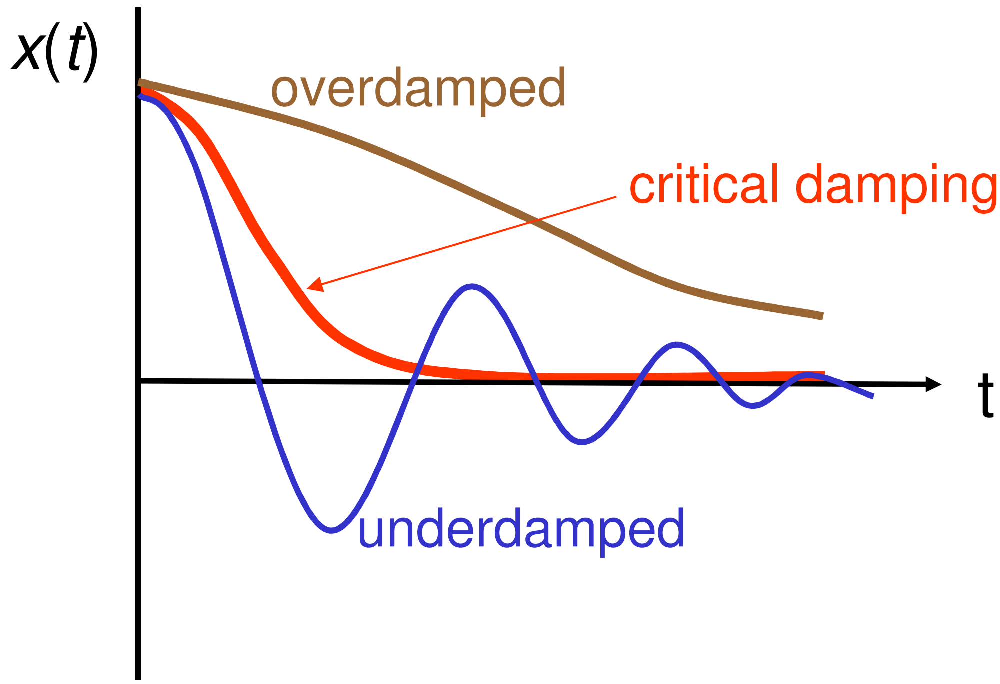


$b > 2m\omega_0$: "Overdamped", no oscillation.

$b < 2m\omega_0$: "Underdamped", oscillations with decreasing amplitude.

$b = 2m\omega_0$: "Critically damped".

Critical damping provides the fastest dissipation of energy.



Example: Automobile suspensions:

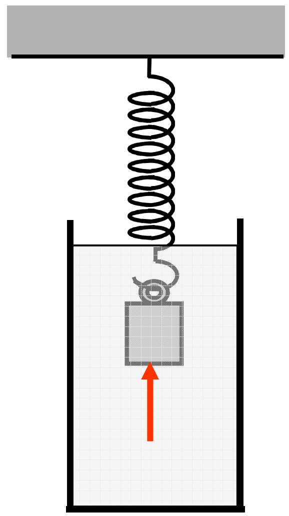
Should they be

a) Underdamped

b) Critically damped

c) Overdamped?

Forced Oscillations



A periodic, *external* force pushes on the mass (in addition to the spring and damping):

$$F_{ext}(t) = F_{\max} \cos \omega t$$

The frequency ω is set by the machine applying the force. The system responds by oscillating at the ***same frequency*** *ω. The amplitude can be very large if the external driving frequency is close to the "natural" frequency of the oscillator.*

$$\omega_0 = \sqrt{\frac{k}{m}}$$ is called the natural frequency or resonant frequency of the oscillation.

Newton's 2nd Law: $$\Sigma F = F_{max} \cos \omega t - kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

same ω

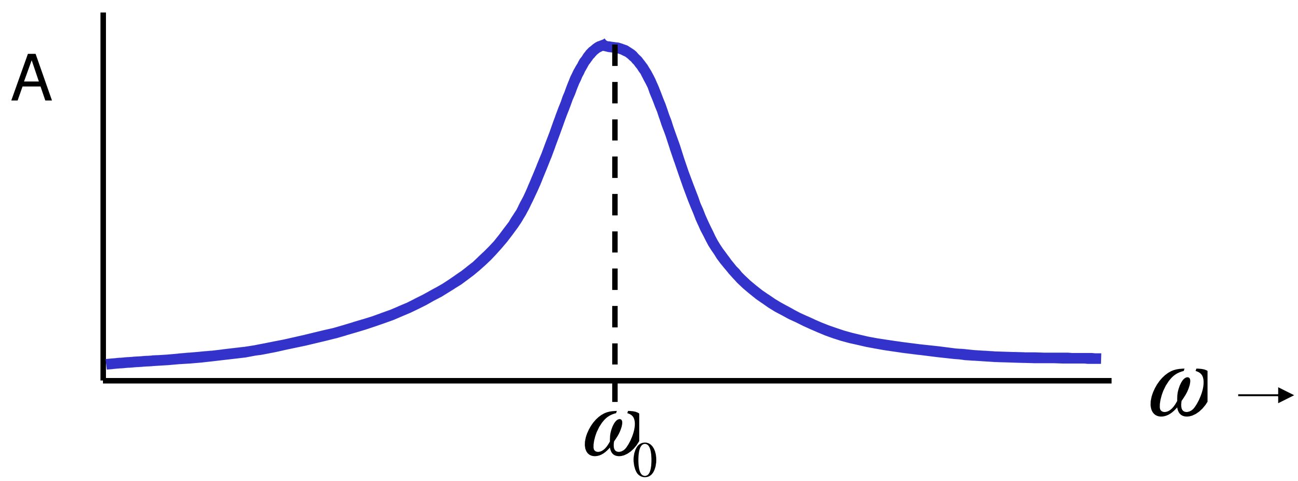
Assume that $x = A\cos(\omega t + \phi)$; then

$$A = \frac{F_{max}/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{\omega b}{m}\right)^2}}$$

$$A = \frac{F_{\max}/m}{\sqrt{\left(\omega^2 - \omega_0{}^2\right)^2 + \left(\frac{\omega b}{m}\right)^2}}$$

So, large A as $\omega \rightarrow \omega_0$

i.e. if the external "push" has the same frequency as the resonant frequency ω_0. The driving force is in resonance with the system.



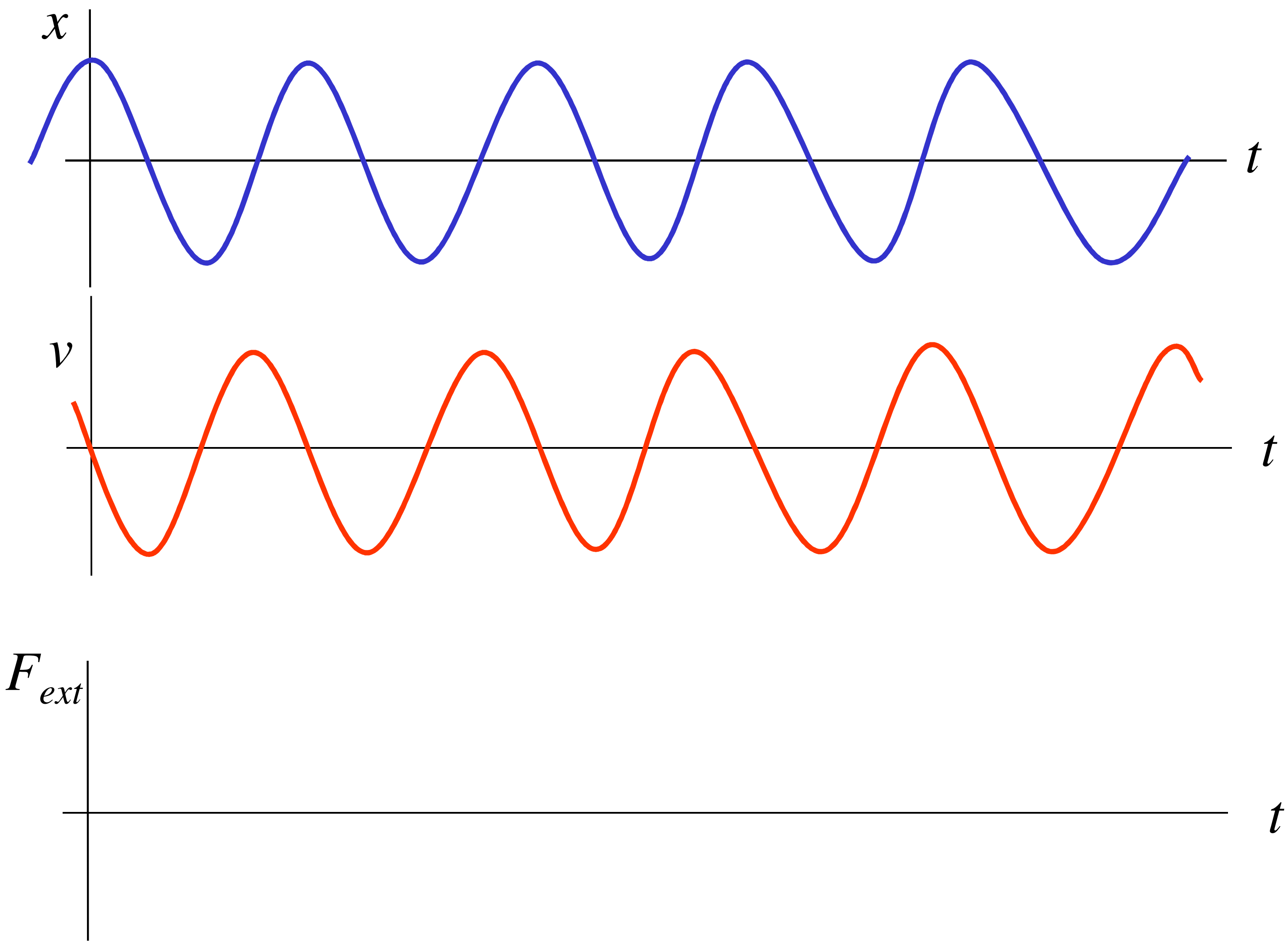
Resonance occurs because the driving force changes direction at just the same rate as the "natural" oscillation would reverse direction, so the driving force reinforces the natural oscillation on every cycle.

Question (about the relative phase): Where in the cycle should the driving force be at its maximum value for maximum average power?

A) When the mass is at maximum x.

B) When the mass is at the midpoint ($x = 0$).

C) It doesn't matter.


x
t
v
t
F
ext
t

Summary

- *For weak damping, the system oscillates, and the amplitude decreases exponentially with time.*
- *With sufficiently strong damping, the system returns smoothly to equilibrium without oscillation.*

- *An oscillator driven by an external periodic force will oscillate with an amplitude that depends on the driving frequency. The amplitude is large when the driving frequency is close to the "natural" frequency of the oscillator.*