XV–Fluid Mechanics

- Definition of Pressure
- Pressure as a function of depth
  - Pascal's Law
- Pressure Measurements
- Buoyant Forces
  - Archimedes's Principle
- Fluid Dynamics
- Streamlines and the Equation of Continuity
- Bernoulli's Equation

Matter (three states)
- Solid (definite volume & shape)
- Fluid (definite volume, not shape)
- Gas (no definite volume & shape)

Fluid: Any substance that can flow, liquids and gases.
(molecules held together by weak cohesive forces and by forces exerted by walls of container.)

Mechanics of fluids
- Static
- Dynamics

Density: mass of unit volume of a substance,

$$\rho = \frac{M}{V} \quad (kg/m^3)$$

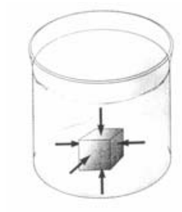
Pressure: ratio of the force to the area over which it is applied,

$$P = \frac{F}{M} \quad (N/m^2 \equiv pa \ (pascal))$$

- Scalar
- Proportional to magnitude of force;

Fluid Characteristics:
- Can not pull.
- Can not shear (Twist).
- Only compress (push).
- Forces by the fluid act always perpendicular to the surfaces.

For small area: $P = \frac{dF}{dA}$



Experience of pressure changes:

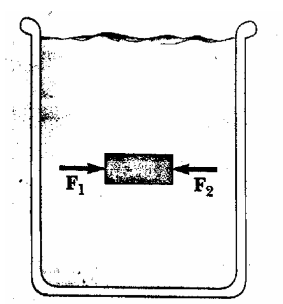
- Step on you in high heels.
- Changes height, Changes in plane.
- Ears popping in driving down "the mountain"
- Ears hurting while diving

Variation of pressure with depth:

- Fluid at rest $\Rightarrow$ Static Equilibrium

a)

$\vec{F}_1 = -\vec{F}_2$



(a)

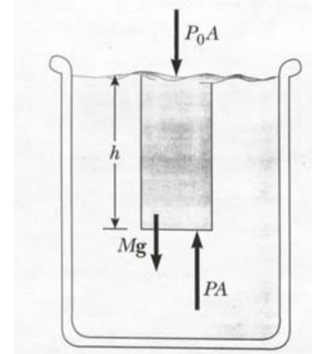
b)

$PA - Mg - P_0 A = 0$

$Mg = \rho V g = \rho A h g$

$PA - \rho h A g - P_0 A = 0$

$P = P_0 + \rho g h$



(b)

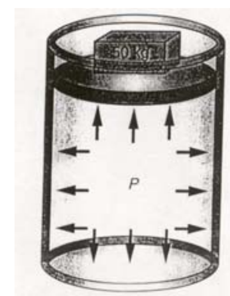
$P_0 = 1.01 \times 10^5\ Pa = 1\ atm \qquad (at\ see\ level)$

Pressure:

- Doesn't depend on the shape of the vessel
- Depends on the height
- Depends on the density.

Pascal law:

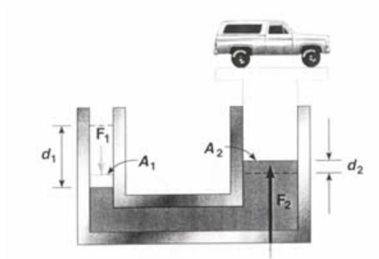
- *A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of container.*



- Any change in the pressure of a fluid is transmitted uniformly in all directions throughout the fluid.

- A small force on small area, can produce a big force over a big area.

Example: Hydraulic jack:



$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1 \frac{A_2}{A_1} \quad >> F_1$$

Like: Hydraulic Brakes, Car Lift, …

Work:

$$W_{F_1} \overset{?}{=} W_{F_2}$$

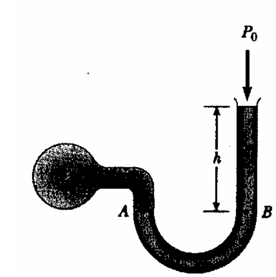
$$A_1 d_1 = A_2 d_2 \quad \Rightarrow \quad \frac{A_2}{A_1} = \frac{d_1}{{}_2}$$

$$F_2 = F_1 \frac{A_2}{A_1} = F_1 \frac{d_1}{d_2}$$

$$F_2 d_2 = F_1 d_1 \quad \Rightarrow \quad W_{F_1} = W_{F_2}$$

Pressure Measurements:

Monometer (Open-tube):

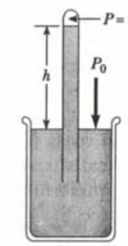


$$P = P_0 + \rho g h$$

$$P - P_0 = \rho g h$$

$$P = P_0 + \rho g h \quad \text{“Absolute Pressure”}$$
$$P - P_0 = \rho g h \quad \text{“Gage Pressure”}$$

Barometer (Closed-tube):



$$P_{0\,(Air)} = (\rho g h)_{Mercury}$$

How about Water Barometer???

$$P_{0\,(Air)} = (\rho g h)_{Water}$$

$$1.013\times10^5 \; N/m^2 = (1000\,kg/m^3)(9.8\,m/s)\,h$$

$$h = 10.3 \; m$$

Buoyant Forces and Archimedes's Principle:

- Buoyant objects: boats, balls, people, …
- Buoyant force: Upward force acts on immersed object by a fluid.

Buoyant Forces:

- *Any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.*

Archimedes's Principle:

- *The magnitude of the buoyant force always equals the weight of the fluid displaced by the object.*

$P_t = \rho g h_t$ ≡ pressure at top

$P_b = \rho_f g h_b$ ≡ pressure at bottom

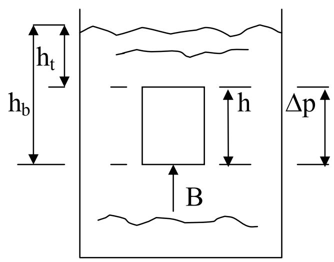
$\Delta P = P_b - P_t = \rho_f g (h_b - h_t)$

$\Delta P = \rho_f g h = B/A$

$B = A\Delta P = \rho_f g h A = \rho_f V g$

$= M_f g = F_g = w_f$

$\therefore \quad B = w_f$



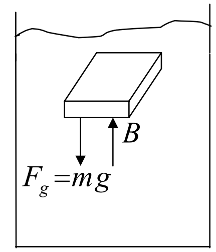
where w_f is the weight of the displace fluid by the object.

$\therefore$ { *magnitude of buoyant force = weight of displaced fluid* }

Totally submerged object:

$$F = ma = B - mg$$
$$= \rho_f g V - \rho_o V g$$

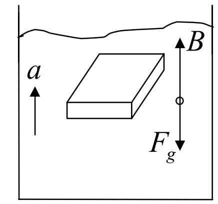
$$F = (\rho_f - \rho_o) V g$$



For $\rho_f > \rho_o$

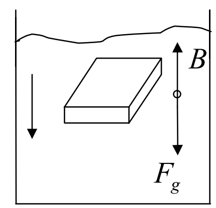
accelerated upward

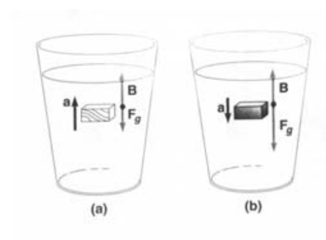
$$F = (\rho_f - \rho_o) V g > 0$$



For $\rho_f < \rho_o$, sink

$$F = (\rho_f - \rho_o) V g < 0$$





Floating Object:

Object of volume $\equiv V_0$

Object is static Equilibrium: $\Rightarrow B = F_g = mg$

Floating = partially submerged

But: $B = \rho_f V_f g$

$V_f \equiv volume\ of\ fluid\ displaced$

$$\Rightarrow \rho_f V_f g = mg = \rho_0 V_0 g$$

$$\frac{V_f}{V_o} = \frac{\rho_o}{\rho_f}$$

$$\frac{V_f}{V_o} \equiv Fraction\ of\ the\ volume\ of\ the\ object\ below\ the\ surface\ of\ the\ fluid.$$

$$F = (\rho_f - \rho_o) V g = 0$$

$$\Rightarrow \rho_f = \rho_o$$

Fluid Dynamics: "*Fluid in motion*"

Flow characteristic: { Laminar / Turbulent }

Laminar (*Steady*) : Each particle of the fluid follows a smooth path, such that the paths of different particle never cross each other.

Turbulent (*irregular*): small whirlpool regions

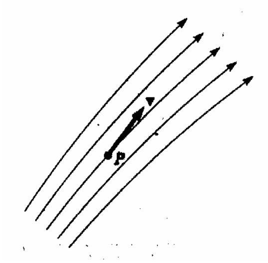
Examples: ?

Some terms:

- *Viscosity:* the degree of friction in the fluid.
- *Viscous force:* resistance to moving two adjacent layer of fluid relative to each other.

Fluid in Motion:

- Streamline: the path taken by a fluid particle in steady flow.



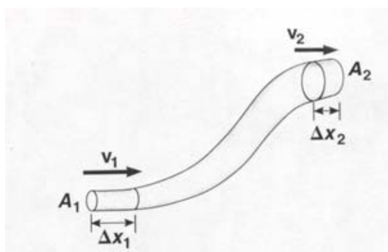
- Fluid Velocity: tangent to the streamline
- Tube of flow: set of streamline.

Ideal Fluid:

- Nonviscous
- Steady
- Incompressible
- Irrotational

Equation of continuity:

- Steady flow



Conservation of mass: $\Delta M_1 = \Delta M_2$

$$\Delta M_1 = \rho \Delta V_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$$
$$\Delta M_2 = \rho \Delta V_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$$

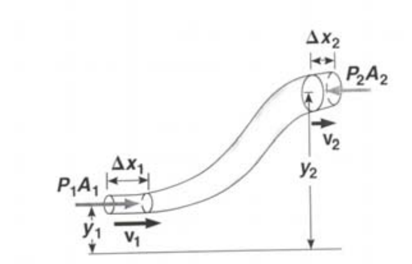
$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$\Rightarrow \quad A_1 v_1 = A_2 v_2 \qquad \textit{Equation of Continuity}$$

Av: is called the *Flow Rate*, or *Volume Flux*.

Bernoulli's Equation:

- Gives the relationship between: fluid speed, pressure, and elevation.



The work done by the force exerted by fluid in section 1:

$$W_1 = P_1 A_1 \Delta x_1 = P_1 V$$

And the work on side 2: $W_2 = - P_2 A_2 \Delta x_2 = P_2 V$

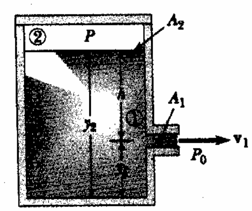
The net force: $W = (P_1 - P_2)\, V$

Using conservation of energy, and the relation between work done by applied force and change of mechanical energy:

$$\Delta W = \Delta K + \Delta U$$
$$P_1 V + \tfrac{1}{2} m_1 v_1^2 + m g y_1 = P_2 V + \tfrac{1}{2} m_2 v_2^2 + m_2 g y_2$$

$$\therefore \quad PV + \tfrac{1}{2} m v^2 + m g y = cons. \qquad \textit{Bernoulli's Equation.}$$

- Application of Bernoulli's Equation:
  - Torricelli's Law



$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_2 = \frac{A_1}{A_2} v_1$$

$$A_2 >> A_1$$

$$\Rightarrow v_2 = \frac{A_1}{A_2} v_1 = 0$$

$$\therefore \; P_0 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_0 + \rho g y_2$$

$$v_1^2 = 2g(y_2 - y_1) = 2g\,yh$$

$$v_1 = \sqrt{2g\,yh}$$

- Another Example:

Figure 11.34 (*a*) Air flowing around an airplane wing. The wing is moving to the right. (*b*) The end of this wing has roughly the shape indicated in part *a*. (© Gary Gladstone/The Image Bank/ Getty Images)