Aspects of Field Theory with Higher Derivatives

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Based on:
Carrillo-González, Masoumi, ARS, & Trodden 1607.05260 (PRD)
and 1703.00909 (PRD), ARS & Trodden 1709.xxxxx

Outline

- Motivation
- Galileon tunneling
- Galileon solitons
- Higher derivatives, modified gravity, and EFTs

Why non-canonical kinetic terms?

- Scalars with **non-canonical kinetic terms** are ubiquitous in particle physics and cosmology, e.g.,
    - Dark energy
    - Modified gravity
    - Supergravity/string theory

Galileons

- Higher-derivative scalars discovered in response to DE problem, starting with DGP
- Properties of the galileons:
  - Second-order equations of motion
  - Galilean invariance $\phi \to \phi + c + b_\mu x^\mu$
  - Vainshtein mechanism: nonlinearities kill "fifth force"
  - Non-renormalization theorem

Galileon Lagrangians

- The five Lagrangians in D=4 consistent with galilean symmetry and with second-order eoms on flat space:

$$\mathcal{L}_2 \sim (\partial\phi)^2 \qquad \sim \varepsilon\varepsilon\partial\phi\partial\phi$$

$$\mathcal{L}_3 \sim (\partial\phi)^2\Box\phi \qquad \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi$$

$$\mathcal{L}_4 \sim (\partial\phi)^2\left[(\Box\phi)^2 - \phi_{\mu\nu}^2\right] \qquad \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi\partial^2\phi$$

$$\mathcal{L}_5 \sim (\partial\phi)^2\left[(\Box\phi)^3 - 3\phi_{\mu\nu}^2\Box\phi + 2\phi_{\mu\nu}^3\right] \qquad \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi\partial^2\phi\partial^2\phi$$

with

$$\phi_\mu \equiv \partial_\mu\phi$$

$$\phi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi$$

Generalizing galileons: without gravity

- Most general flat-space scalars with second-order eoms:

$$f_n(\phi, (\partial\phi)^2) \times \mathcal{L}_n$$

- Lose galilean invariance (though may have other interesting symmetries)

- Some special cases—DBI, conformal, and (A)dS galileons—have interesting origins in higher-dimensional physics

- **Vainshtein mechanism**! How does this affect nonperturbative solutions?

Generalizing galileons: with gravity
Horndeski

- Generalizing to include gravity yields **Horndeski gravity**, the most general scalar-tensor theory with second-order equations of motion ($X=(\partial\phi)^2$):

$$\mathcal{L}_2 = G_2(\phi, X),$$
$$\mathcal{L}_3 = G_3(\phi, X)\Box\phi,$$
$$\mathcal{L}_4 = G_4(\phi, X)R - 2G_{4,X}\left[(\Box\phi)^2 - \phi_{\mu\nu}^2\right],$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}\left[(\Box\phi)^3 - 3\phi_{\mu\nu}^2\Box\phi + 2\phi_{\mu\nu}^3\right]$$

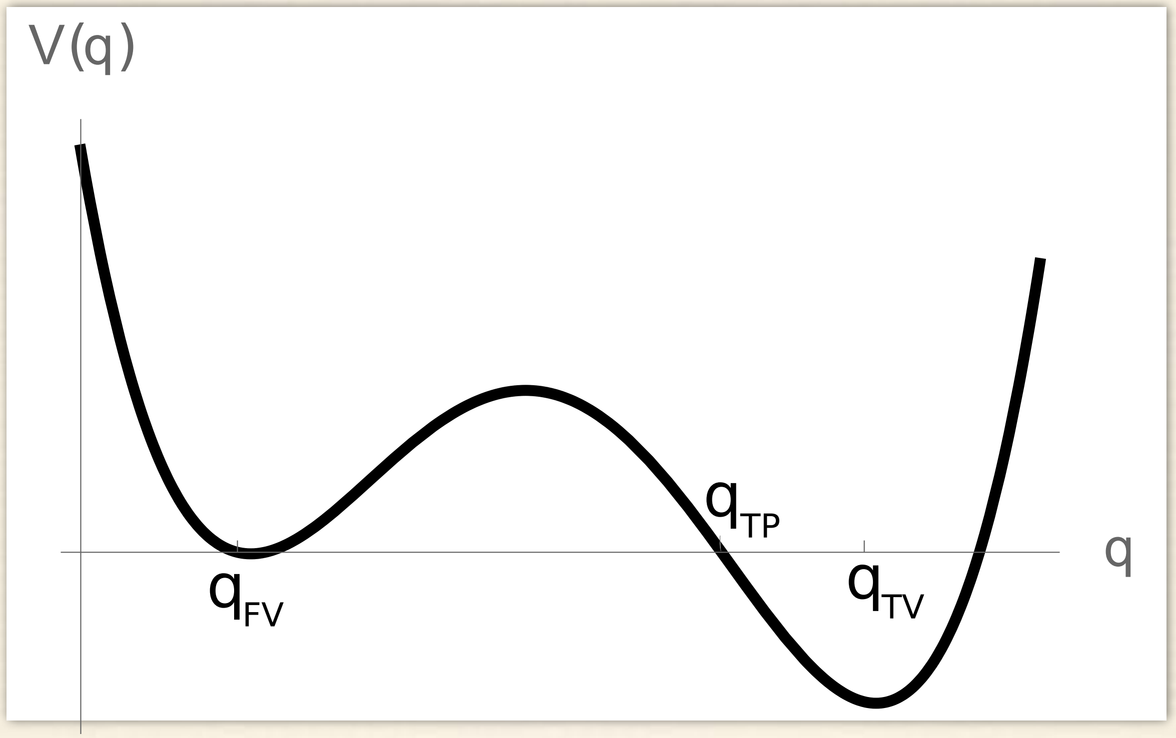
- These Lagrangians are now ubiquitous in modified gravity

Such theories have interesting non-perturbative physics

- Long known that canonical scalars have a zoo of **interesting nonlinear phenomena**
- This talk:
  - Quantum tunneling
  - Solitons
- Both arise when potentials have **non-degenerate minima**
- These have been well-understood for decades. What changes when we introduce newly-discovered kinetic structures?

Tunneling

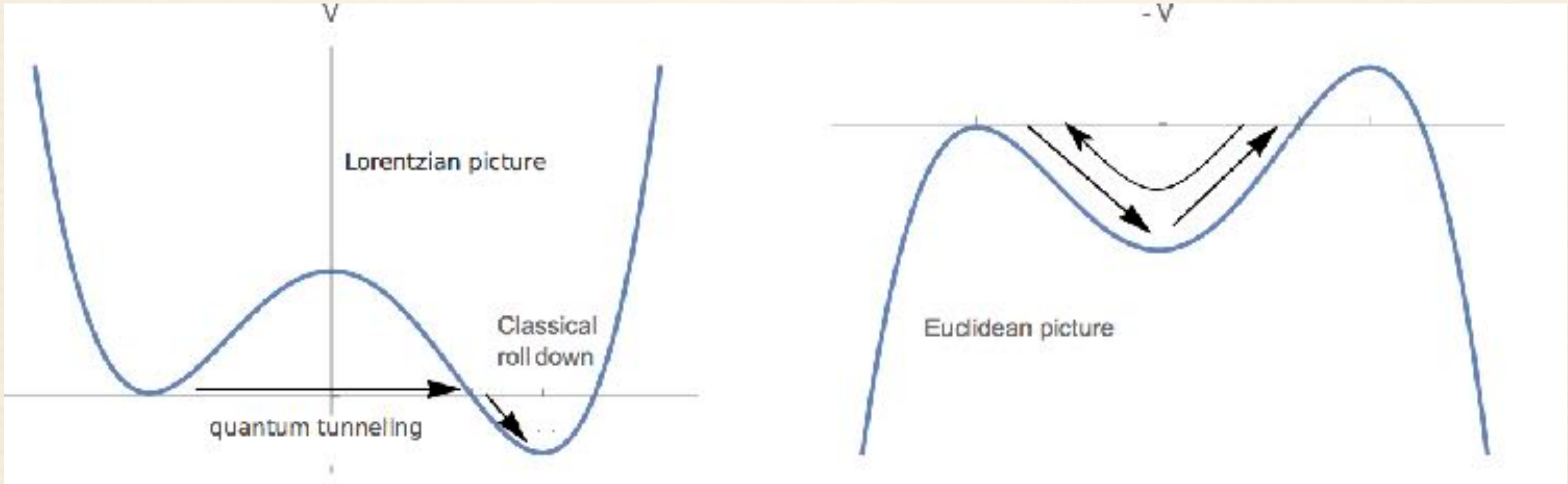
- Consider a potential with two minima at different heights:



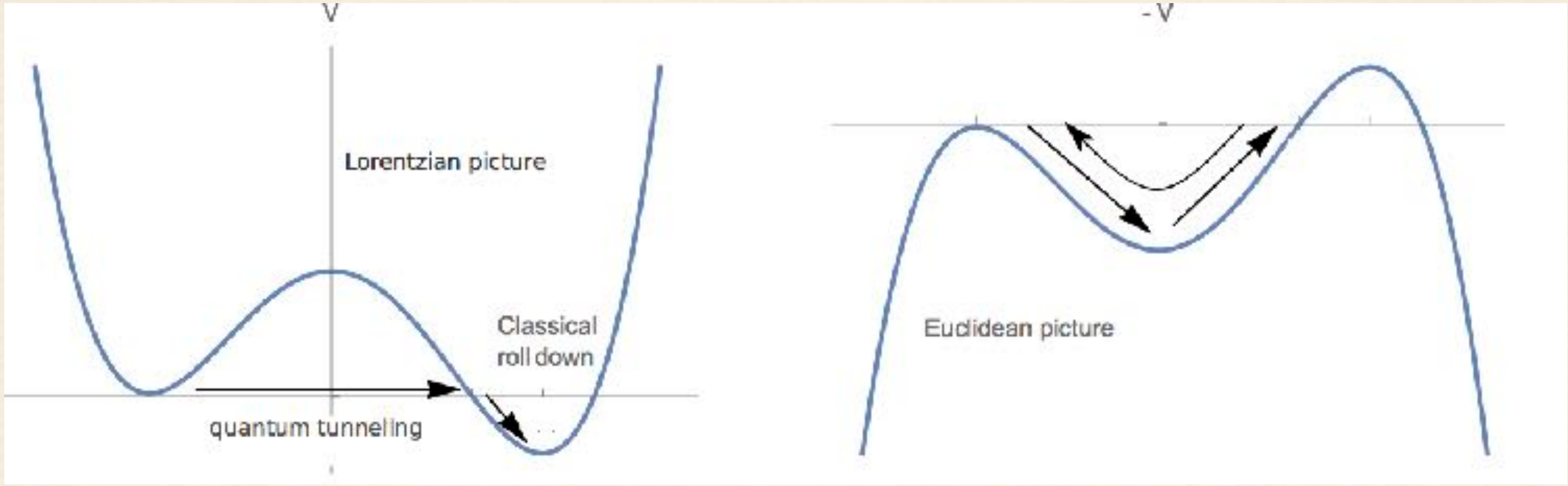
- Classically both minima are stable, but quantum mechanics induces **decay** of false vacuum via tunneling

Tunneling: Lorentzian and Euclidean pictures

- Prescription for determining decay rate (Coleman 1977):
- Transform to **Euclidean time**, i.e., invert potential



Tunneling: Computing the decay rate



- The **action** of the Euclidean "bounce" solution determines the decay rate:

$$\frac{\Gamma}{V} \sim e^{-B}$$

$$B = S_{\rm E}(\text{bounce}) - S_{\rm E}(\text{FV})$$

Does WKB hold with non-canonical kinetic terms?

Details: arXiv:1703.00909

- The result $\Gamma/V \sim e_{-B}$ was proven for the canonical scalar field using WKB approximation
- By solving for the wavefunctional $\psi[\phi]$ in the semi-classical limit, we show that for general

$$L = L(\phi, \dot{\phi}, \partial_i \phi, \partial_i \partial_j \phi)$$

  the dominant contribution to the decay rate comes from the solution to the Euclidean equation of motion
- **Explicitly see: non-canonical kinetic terms do not change Coleman prescription for decay rate***
  - *Assumption: second-order eoms

Decay rates

- Problem of finding decay rate Γ amounts to solving Euclidean eoms with O(4) symmetry
- Warm up: $L = P(X) + V(\phi)$ with $X=(\partial\phi)^2$. Euclidean action:

$$S_{\mathrm{E}} = 2\pi^2 \int \rho^3 (P + V) \mathrm{d}\rho$$

- Define **non-standard Lagrangian** with volume factor removed:

$$S_{\mathrm{E}} \equiv 2\pi^2 \int \rho^3 L \mathrm{d}\rho$$

and similarly non-standard canonical momentum:

$$\pi_\phi \equiv \frac{\partial L}{\partial \dot{\phi}}$$

P(X) decay rates in the thin-wall limit

- Consider **thin-wall limit** (small potential difference between the two minima):

$$\epsilon \equiv V_{\rm FV} - V_{\rm TV} \ll V$$

- The bounce factor in this limit is

$$B = \frac{27\pi^2 S_1^4}{2\epsilon^3}$$

- with S_1 the tension of the bubble wall,

$$S_1 \equiv \int_{\rm wall} \pi_\phi {\rm d}\phi$$

- In the canonical case P(X) = X/2 this reduces to Coleman's famous result ✓
- *However*, $\Gamma/V \sim e_{-B}$ depends extremely sensitively on even small changes in S_1

Galileon decay rates

- Finally consider **galileons** (incl. non-gravitational generalizations). For concreteness consider cubic galileon,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi - V(\phi)$$

- Two regimes:
  - **Standard**: Canonical kinetic term dominates, standard decay rate:

$$B_{\rm can} = \frac{27\pi^2(S_1^{\rm can})^4}{2\epsilon^3}, \qquad S_1^{\rm can} = \int_{\rm wall}\dot{\phi}\mathrm{d}\phi$$

  - **Vainshtein**: Galileons dominate, qualitatively different decay rate:

$$B_{\rm gal} = \frac{2\pi^2(S_1^{\rm gal})^2}{\epsilon} \qquad S_1^{\rm gal} = \int_{\rm wall}\frac{6}{\Lambda^3}\dot{\phi}^2\mathrm{d}\phi$$

Galileon decay rates: Vainshtein mechanism

- Which regime we're in depends on free parameters:
  - ε: difference between potential heights
  - Δϕ: difference between location of the two minima
  - Λ: energy scale associated to the galileon
- Canonical(/galileon) term dominates if $\frac{\epsilon}{\Delta\phi} \gg (\ll)\ \Lambda^3$
- Equivalently: depends on whether Euclidean bubble size ρ is larger or smaller than a threshold value, akin to a **Vainshtein radius**
- If the galileon dominates, the decay rate can be **many orders of magnitude** larger than for a canonical scalar with the same potential

Solitons

- Solitons are:
  - Non-trivial field configurations
  - Finite energy
  - Localized in space
  - Do not dissipate
- Exist due solely to **nonlinearities in the field**, no external source
- E.g., if two non-degenerate minima, can have **domain wall**

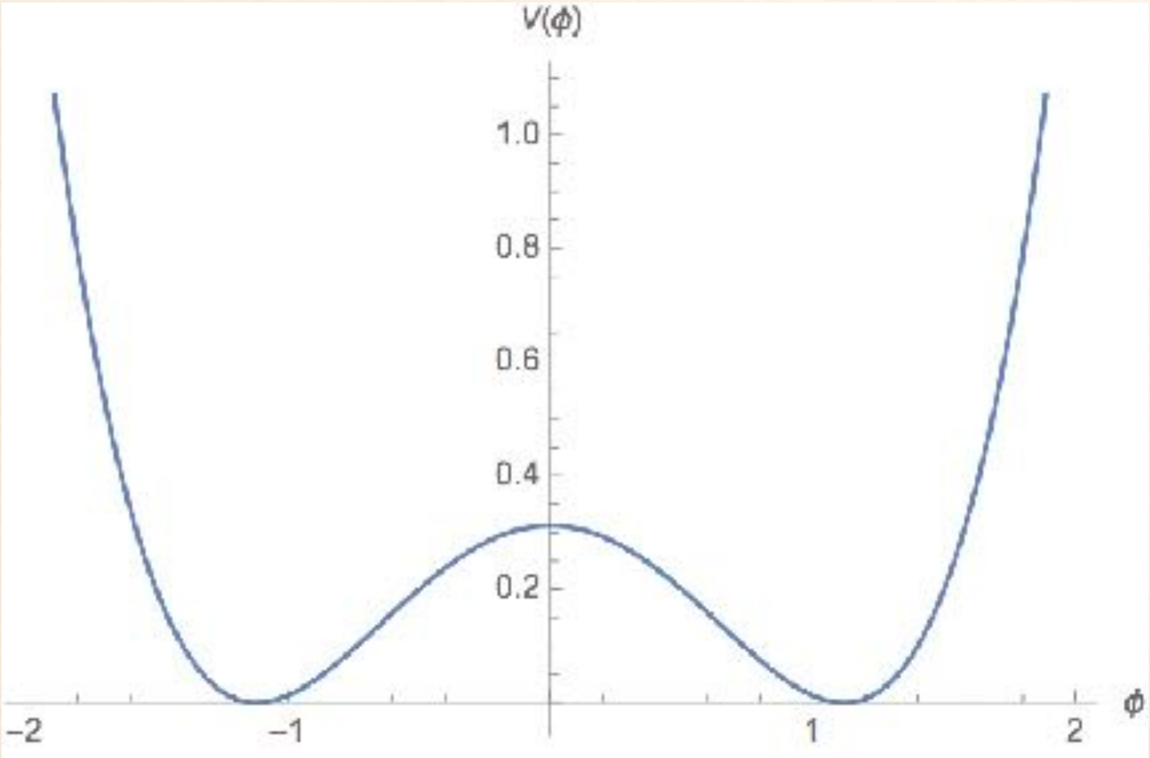
Domain walls

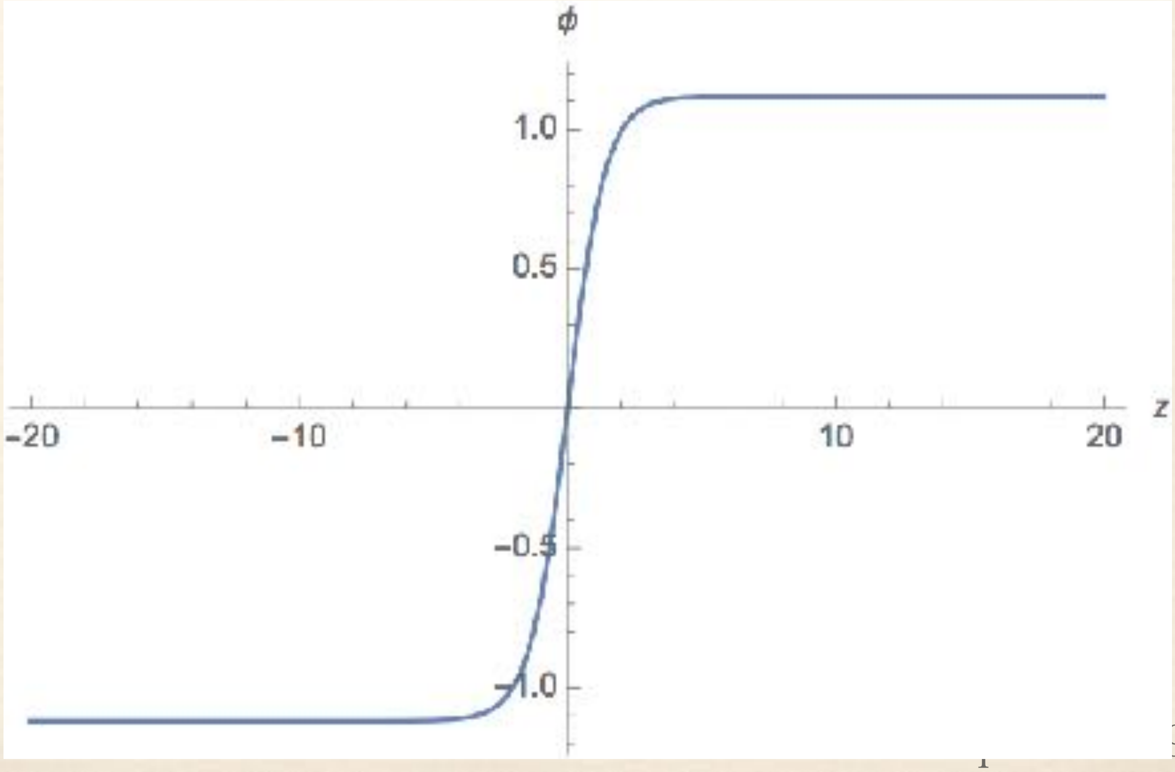
- For example,

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

is solved by

$$\phi(z) = \frac{m}{\sqrt{\lambda}}\tanh\left(\frac{mz}{\sqrt{2}}\right)$$





Derrick's theorem: a no-go

- Zero-mode/scaling arguments (Derrick's theorem) tell us we need **a potential** or **time dependence**.
  - Derrick's theorem extended to galileons: Endlich et al. arXiv:1002.4873
  - Extended to generalized galileons: us
- Let's focus on each of these in turn.

Adding a potential

Details: arXiv:1607.05260

- Stable static walls difficult to obtain:
  - Standard galileons with a potential: galileon Lagrangians are total derivatives in 1D, so no difference from canonical case
  - Conformal galileons with a potential: domain walls do not exist
  - (A)dS galileons: naturally possess a potential due to their symmetries. Leads to bubbles, but they shrink and suffer from ghosts

Time dependence—light-speed domain walls

- Time dependence requires v=c so that there isn't a frame where the soliton is at rest
- Masoumi & Xiao, arXiv:1201.3132—standard galileons have stable light-speed solitons
- Generalized galileons:
  - DBI galileons: stable solitons ✓
  - Conformal galileons: unstable **X**
  - (A)dS galileons: naturally include a potential, so should not include solutions with v=c

Summary of nonperturbative effects

- Tunneling rates very sensitive to kinetic structure, especially for galileons outside a Vainshtein region
- Non-canonical kinetic terms can severely boost vacuum decay rate
- Difficult to construct static galileon domain walls
- Easier for walls moving at speed of light

Effective field theory and modified gravity: All* theories are effective theories

- **Decoupling**: high-energy physics not needed to understand phenomena at low energies
- EFT approach underlies our ability to use nonrenormalizable theories, including GR
- The lens through which modern physical theories are viewed!
- **Unless you have a theory of everything at hand, new theories should be seen as EFTs**

What does (and doesn't) go into an EFT

- Write down most general Lagrangian consistent with particle content and symmetries, expanded in powers of E/M
  - M: some (high) energy scale signaling breakdown of EFT
- Requirements: locality, analyticity, etc.
- Notably **not** required: second-order equations of motion

Modified gravity as an EFT

- Modifications of GR: useful for dark energy, dark matter, inflation, etc.
- These theories are nonrenormalizable, hence should be seen as effective theories
    - Construct MG analogously to beyond-Standard Model physics
- Most MG theories are constructed to be **ghost-free**: Horndeski, Lovelock, generalized galileons, etc.
- Higher derivatives are generic in EFTs — healthy UV does **not** imply ghost-free EFT

Questions

- When should (and shouldn't) we restrict to ghost-free theories like Horndeski and Lovelock?
- What new higher-derivative terms can one write down?
- Are these phenomenologically interesting?

Healthy UV can have unhealthy EFT
I. Is quantum gravity unstable?

- GR is lowest-energy term in EFT expansion:

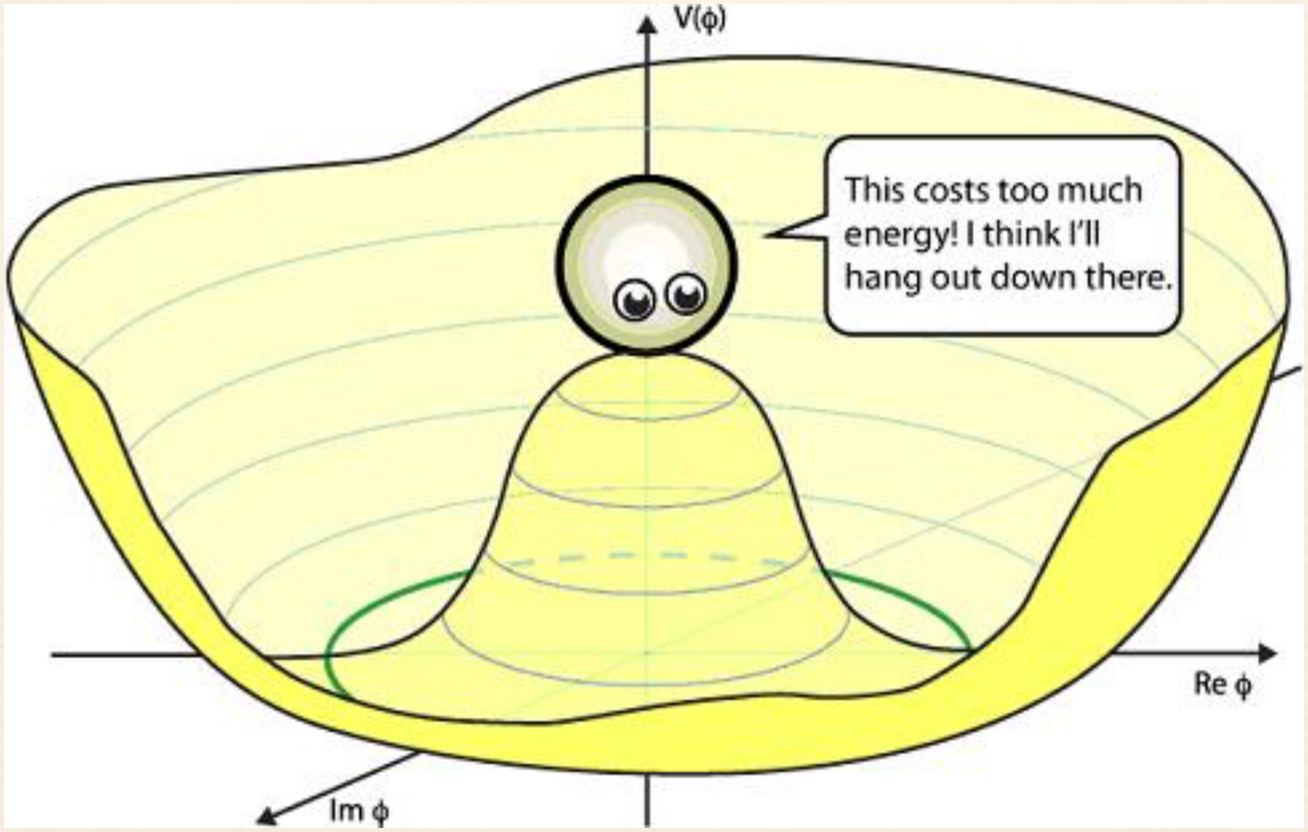
$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \frac{d_1}{M^2}R^3 + \cdots$$

- All terms (besides R^n) lead to ghosts
- **Is flat space unstable in quantum gravity?**
- Answer (fortunately) is no

Healthy UV can have unhealthy EFT
II. U(1)-breaking scalar

- Illustrative example (Burgess and Williams 1404.2236):

$$S = \int \mathrm{d}^4x \left[-\frac{1}{2}\partial_\mu \Phi^\star \partial^\mu \Phi - V(\Phi^\star \Phi) \right]$$



Healthy UV can have unhealthy EFT
II. U(1)-breaking scalar

- Writing

$$\Phi(x) = \frac{v}{\sqrt{2}}\,(1+\rho(x))\,e^{i\theta(x)}$$

we have a massless Goldstone θ and a massive ρ with $M^2=\lambda v^2$

- The action becomes

$$\mathcal{L} = \int \mathrm{d}^4x \left[-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho)\right]$$

Healthy UV can have unhealthy EFT
II. U(1)-breaking scalar

$$\mathcal{L} = \int \mathrm{d}^4x \left[-\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(1+\rho)^2(\partial\theta)^2 - V(\rho) \right]$$

- For energies << M, we can **integrate out** ρ to obtain an effective action for θ

- Solution to ρ eom is highly nonlocal, but can localize by writing perturbatively in 1/M

$$\Box\rho - (1+\rho)(\partial\theta)^2 - V' = 0 \quad \Longrightarrow \quad \rho = -\frac{(\partial\theta)^2}{M^2} - \frac{(\partial\theta)^4 + 2\Box(\partial\theta)^2}{2M^4} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

Healthy UV can have unhealthy EFT
II. U(1)-breaking scalar

- At low energies, EFT for angular mode θ is

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu\rho}\theta^{,\nu}\theta_{,\rho} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

- $O(M_{-4})$ term is **ghostly**!
- Original theory is healthy — what happened?
- Resumming full 1/M expansion (nonlocal) would cure ghost — higher derivatives are an artifact of truncation
- EFT point of view: ghost is of mass ~M, cannot be produced within EFT

Ghostbusting

- EFTs with higher derivatives require additional initial conditions — **spurious** solutions
- Only a subset of these are **physical** insofar as they reflect solutions of the full theory
- Ghost instabilities associated to higher derivatives are **not present** in physical solutions

Ghostbusting

- Given a higher-derivative EFT, how do we identify these physical solutions?
  - Simply solving the equations of motion will lead to disaster
- Are they (exact) solutions to some other, ghost-free theory?
  - If yes: justifies use of ghost-free theories
  - If no: opens up theory space

How do we identify physical solutions?

- Consider in 1D particle mechanics

$$L = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\epsilon\ddot{x}^2 + \mathcal{O}(\epsilon^2)$$

  the eom is

$$\ddot{x} - \epsilon x^{(4)} = 0$$

  and has solutions

$$x = A + Bt + Ce^{t/\epsilon} + De^{-t/\epsilon}$$

- Exponential solutions are bad but not physical—not consistent with perturbative expansion in ε
- Only the C=D=0 solutions are **physical** insofar as they perturbatively reflect solutions of the full (UV) theory

Reduction of order method

- Instead of setting initial conditions to remove runaways (numerically unstable?), can **reduce order** of EFT eom.

- In this example, defining $E \equiv \ddot{x} - \epsilon x^{(4)} = \mathcal{O}(\epsilon^2)$ we see that, to order in which we're working,

$$E + \epsilon \ddot{E} = \ddot{x} + \mathcal{O}(\epsilon^2) \quad \Longrightarrow \quad \ddot{x} = \mathcal{O}(\epsilon^2)$$

  so that the physical C=D=0 solutions emerge naturally

- Allows to straightforwardly deal with higher derivatives in EFTs, e.g., numerically

When do the physical solutions correspond to an action?

- Key: **field redefinitions**. Recall the U(1) scalar:

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu\rho}\theta^{,\nu}\theta_{,\rho} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

and send

$$\theta \to \theta + \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu}\theta^{,\nu}$$

Leaves us with a second-order theory, the quartic galileon!

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 + \frac{1}{M^4}(\partial\theta)^2\left[\theta_{,\mu\nu}\theta^{,\mu\nu} - (\Box\theta)^2\right] + \mathcal{O}\left(\frac{1}{M^6}\right)$$

When do the physical solutions correspond to an action?

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 - \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu\rho}\theta^{,\nu}\theta_{,\rho} + \mathcal{O}\left(\frac{1}{M^6}\right)$$

$$\theta \to \theta + \frac{2}{M^4}\theta_{,\mu\nu}\theta^{,\mu}\theta^{,\nu}$$

$$\mathcal{L} = -\frac{1}{2}(\partial\theta)^2 + \frac{1}{2M^2}(\partial\theta)^4 + \frac{1}{M^4}(\partial\theta)^2\left[\theta_{,\mu\nu}\theta^{,\mu\nu} - (\Box\theta)^2\right] + \mathcal{O}\left(\frac{1}{M^6}\right)$$

- The problematic higher derivatives have been shunted off to $O(M^{-6})$, which we can safely ignore
- Physical solutions to this EFT could be obtained by exactly solving a quartic galileon (up to $O(M^{-4})$)

Prescription for identifying genuine higher derivatives

- Construct EFT operator basis up to two redundancies: **Integrations by parts** and **field redefinitions (equations of motion)**
  - This is very well understood in beyond-SM physics
- Extra ingredient for modified gravity: construct operator bases with **as few higher derivatives as possible**
- Those higher-derivative terms that are left **should** be included!
- Deal with these by solving equations of motion **perturbatively**

Shift-symmetric scalar EFT basis: no ghosts until dimension 12!

Dimension	Operators
4	$X = (\partial\phi)^2$
5	*None*
6	*None*
7	Cubic galileon*
8	X^2
9	*None*
10	Quartic galileon
11	*None*
12	X^3, $(\phi_{\mu\nu}^2)^2$

Modified gravity EFT: scalar-tensor operator basis

- Field redefinitions enforce lowest-order equations of motion. Scalar-tensor operators which are unaffected should involve **Riemann couplings**

- Consider scalar-tensor EFT in derivative expansion (e.g., Weinberg's EFT of inflation)

Derivatives	Operators
4	X^2, Gauss-Bonnet [Weinberg]
6	X^3, quartic Horndeski, *five new higher-derivative operators*

Six-derivative scalar tensor operators generated through Riemann couplings

$$R_{\mu\nu\alpha\beta}\phi^{\mu\alpha}\phi^{\nu\beta}, \quad R_{\mu\nu\alpha\beta}\phi^{\mu}\phi^{\alpha}\phi^{\nu\beta}, \quad XR^2_{\mu\nu\alpha\beta},$$
$$R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu}, \quad (\nabla R_{\mu\nu\alpha\beta})^2$$

- Should be considered alongside comparable-size Horndeski terms in, e.g., inflation, dark energy. *No reason* a priori to ignore them
- Must deal with higher derivatives in one of two ways:
  - Solve equations of motion order by order, or
  - Reduce order of equations of motion using perturbative nature

Take-home message

- **Advocate for consideration of modified gravity as an EFT, including allowing higher derivatives**
- Healthy UV does not imply ghost-free low-energy EFT
- Additional justification required to restrict to theories with second-order equations of motion
- Both Horndeski and non-Horndeski terms should be treated in EFT expansion
    - Not impossible — e.g., DBI