Superfluids and the Cosmological Constant Problem

Adam R. Solomon
Carnegie Mellon University

With Justin Khoury & Jeremy Sakstein (UPenn)
arXiv:1805.05937 (JCAP)

Outline

1. Intro to the cosmological constant problem(s)
2. Proposed solutions and obstructions
3. Our proposal: finite-temperature superfluid/ Lorentz-violating massive gravity

“The cosmological constant problem is the unwanted child of two pillars of twentieth century physics: quantum field theory and general relativity.”

Tony Padilla

What is the cosmological constant?

- Einstein's equations allow a cosmological constant:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu}$$

- Most pronounced effect is on long distances/cosmology:
  - $\Lambda > 0$: late-time acceleration
  - $\Lambda < 0$: collapse
- Observations indicate the Universe is *accelerating*, consistent with a cosmological constant of size

$$\Lambda \sim \mathcal{O}\left(\frac{\mathrm{meV}^4}{M_{\rm Pl}^2}\right)$$

What could produce Λ?

- One known theoretical source of Λ: *vacuum energy*
  - Equivalence principle: the vacuum gravitates
  - Lorentz invariance: $T_{\mu\nu} \sim \frac{\rho_{\rm vac}}{M_{\rm Pl}^2} g_{\mu\nu}$
- Massive fields contribute $\rho_{\rm vac} \sim m^4$
- Observed Λ is about the vacuum energy due to neutrino masses, tiny compared to any other particles in Standard Model (and beyond)
  - Electron vacuum energy alone would lead to de Sitter horizon ~ 10^6 km
  - Pauli: the radius of the world "nicht einmal bis zum Mond reichen würde" (would not even reach the Moon!)

Where is the vacuum energy?

- The problem is even worse with more particle species:

$$\frac{\rho_{\rm vac,electron}}{\rho_{\rm vac,obs}} \sim 10^{32}$$

$$\frac{\rho_{\rm vac,SM}}{\rho_{\rm vac,obs}} \sim 10^{54}$$

$$\frac{\rho_{\rm vac,Planck}}{\rho_{\rm vac,obs}} \sim 10^{121}$$

- "The worst theoretical prediction in the history of physics!"

Cosmological constant problems old and new

- Cosmic acceleration poses two (logically distinct?) problems:
- "Old problem": why does an enormous vacuum energy not gravitate?
- "New problem": why is there some residual acceleration anyway?
- Often treated separately! Solve one while ignoring the other
- This talk: focus on the **old problem**

Approaches to the old problem

- There are *many* proposed solutions. I will discuss a small sample. For comprehensive lists, see reviews by
    - **Weinberg, Rev. Mod. Phys. 61 (1989) 1-23**
    - Nobbenhuis, gr-qc/0411093
    - Martin, 1205.3365
    - Burgess, 1309.4133
    - Padilla, 1502.05296

Some approaches

- Anthropics: if Λ were bigger, we wouldn't be around to remark on it
  - ¯_(ツ)_/¯
  - Could follow from string landscape + eternal inflation
  - Dimopoulos: danger of "premature application"
- Modifications of gravity: leave Λ alone, but change how it gravitates
  - Degravitation: weaken gravitational response to long-wavelength sources
  - Self-tuning: introduce new field(s) which dynamically counteract Λ

Self-tuning and our modest goal

- We will set a modest goal: field equations solved by Minkowski for arbitrary Λ
- Necessary but not sufficient condition for solving old CC problem
- Other criteria:
    - UV insensitivity, radiative stability, no pathologies, agreement with experiments, reproduce observed cosmological history, etc.!
- Self-tuning runs into a famous obstruction due to Weinberg

Weinberg's famous no-go theorem

- Weinberg (1988); see also Padilla review 1502.05296
- Assume some fields ϕ^A "eat up" vacuum energy,

$$T^{\phi}_{\mu\nu} = -T^{\Lambda}_{\mu\nu}$$

- Assume *Poincaré-invariant* vacua,

$$\phi^A = \text{const}, \qquad g_{\mu\nu} = \eta_{\mu\nu}$$

Weinberg's famous no-go theorem: two paths

- Add a potential: $\mathscr{L} = -\sqrt{-g}(V(\phi^A) + 2\Lambda) + \text{derivatives}$
  - Fine-tuning: only cancels out one specific value of Λ
- Scaling symmetry: $\mathscr{L} = -\sqrt{-g}V_0 e^{4\tilde{\phi}} + \text{derivatives}$
  - Particle masses also vanish; not physical

Evading Weinberg's theorem

- Any no-go theorem has assumptions, pointing the way forward

- Our (and others') approach: *break Poincaré invariance*

$$\Phi^A = \Phi^A(x^\mu), \quad g_{\mu\nu} = \eta_{\mu\nu}$$

- *e.g.*, in cosmology, we might have fields with time dependence

- This implies, at a minimum, that the fields must be accompanied by derivatives

- To leading order in EFT, one derivative per field

Warmup: one scalar

- Can we degravitate with a single scalar?
  - **No.**
  - Why?
- At leading order in derivatives, most general action is

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 P(X)\right], \quad X \equiv (\partial\Phi)^2$$

- NB: this is the EFT of a zero-temperature superfluid

- The total stress tensor is

$$\frac{1}{M_{\rm Pl}^2}T_{\mu\nu} = \frac{1}{2}m^2\left(Pg_{\mu\nu} - 2P_X\partial_\mu\Phi\partial_\nu\Phi\right) - \Lambda g_{\mu\nu}, \quad P_X \equiv \frac{\partial P}{\partial X}$$

- In order to have flat solutions for arbitrary Λ, this must vanish:

1. $P_X = 0$

2. $m^2 P = 2\Lambda$

- The latter requires *fine-tuning.*

- *e.g.*, ghost condensate, $P(X) = X + \lambda X^2/2$

  - Solution: $\Phi = \lambda^{-1/2}t$. But $m^2 P = 2\Lambda$ only if λ is carefully tuned against Λ!

$$\lambda = -\frac{m^2}{4\Lambda}$$

Four scalars

- Fine-tuning can be alleviated with four scalars Φ^A
  - Can eliminate problem of inhomogeneous term in stress tensor
- Consider a simple (and trivially wrong) model:

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda - m^2\eta_{AB}\partial_\mu\Phi^A\partial^\mu\Phi^B\right]$$

- The stress tensor no longer has an inhomogeneous term:

$$\frac{1}{M_{\rm Pl}^2}T_{\mu\nu} = \frac{1}{2}m^2\left(-\eta_{AB}\partial_\alpha\Phi^A\partial^\alpha\Phi^B g_{\mu\nu} + 2\eta_{AB}\partial_\mu\Phi^A\partial_\nu\Phi^B\right) - \Lambda g_{\mu\nu}$$

- This admits flat solutions $\Phi^A = \alpha x^A$, with

$$\alpha = \frac{\sqrt{-\Lambda}}{m}$$

- Degravitates any negative Λ without fine-tuning!
  - Key: instead of tuning theory parameters against Λ, tune *integration constant* α to Λ
  - Tuning is achieved *dynamically*

- Fatal problem: Φ^0 is a **ghost**

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda - m^2\eta_{AB}\partial_\mu\Phi^A\partial^\mu\Phi^B\right]$$

$$= \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2(\partial\Phi^0)^2 - m^2\sum_{i=1}^{3}(\partial\Phi^i)^2\right]$$

- Direct result of internal Lorentz symmetry, which is what we used to remove inhomogeneous term!
  - Blessing and a curse
- Can we self-tune without a ghost?

Ghosts and massive gravity

- The ghost is easy to understand if we recognize this is a theory of **massive gravity**

- Why? Consider adding to GR a mass term of the form

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda - m^2 g^{\mu\nu}\eta_{\mu\nu}\right]$$

- This breaks diff invariance due to $\eta_{\mu\nu}$. Can restore diffs by introducing Stückelberg fields Φ^A,

$$\eta_{\mu\nu} \to \eta_{AB}\partial_\mu\Phi^A\partial_\nu\Phi^B$$

- And we recover the action discussed in the previous slides

Massive gravity and degravitation

- This connects to an old and venerable story:
  - Massive graviton → finite range of gravity → gravity acts as "high-pass filter" screening out sources with wavelengths >> m^{-1}
  - Λ is infinite-wavelength source!
- Massive gravity at linear level: Fierz-Pauli (1939) $h_{\mu\nu}h^{\mu\nu} - (h^{\mu}{}_{\mu})^2$
  - Other linear mass terms have ghosts, just like the example we discussed

Massive gravity and degravitation

- Non-linear: another ghost! (Boulware-Deser, 1972)
- Unique non-linear, ghost-free, Lorentz-invariant massive gravity: de Rham-Gabadadze-Tolley (dRGT, 2010)
- dRGT cannot degravitate large Λ without violating solar system tests of GR (1010.1780)
- This means we cannot use a theory of the form

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 U(\partial_\mu \Phi^A, \eta_{AB}, \varepsilon_{ABCD})\right]$$

Is Lorentz invariance too strong a requirement?

- For cosmology, we only need SO(3), not SO(3,1)
- Idea: break internal boosts

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 U(\partial_\mu \Phi^0, \partial_\mu \Phi^i, \delta_{ij}, \varepsilon_{ijk})\right]$$

- Aim: use newfound freedom to avoid ghosts (and other pathologies) while retaining degravitation
- We will analyze this theory for physically sensible degravitating models with

$$\Phi^0 = \alpha t, \quad \Phi^i = \beta x^i \quad \text{such that} \quad T^\Phi_{\mu\nu}|_{g=\eta} = M_{\rm Pl}^2 \Lambda \eta_{\mu\nu}$$

Interpreting our theory

- Two physical interpretations of this type of theory:

  1. Lorentz-violating massive gravity

  2. Low-energy EFT of self-gravitating fluid

- Difference hinges only on coordinate choice

Lorentz-violating massive gravity as a fluid EFT

- Consider the fields $\Phi^A = \Phi^A(t,x)$ to be comoving (Lagrangian) coordinates of a fluid
- Fluid rest frame is a coordinate system in which $\Phi^A = \alpha x^A$
- EFT describing excitations of fluid is a derivative expansion in Φ^A obeying any relevant symmetries

Building blocks and symmetry

- At leading order in derivatives, action is built out of

$$C^{AB} \equiv g^{\mu\nu}\partial_\mu\Phi^A\partial_\nu\Phi^B \quad \Longrightarrow \quad \mathscr{L} = U(C^{00}, C^{0i}, C^{ij})$$

- Choice of operators determines symmetry-breaking pattern and hence fluid, e.g.,
  - Solids: $\mathscr{L} = U(C^{ij})$
  - Zero-temperature superfluids: $\mathscr{L} = U(C^{00})$
  - Finite-temperature superfluids:

$$\mathscr{L} = U(C^{00}, \det C^{ij}, \det C^{AB})$$

The importance of coordinates

- If we move to the fluid rest frame, $\Phi^A = \alpha x^A$, then we recover the Lorentz-violating massive gravity picture:

$$U(C^{00}, C^{0i}, C^{ij}) \xrightarrow{\Phi^A = \alpha x^A} U(h_{00}, h_{0i}, h_{ij})$$

- We refer to this as unitary gauge
- e.g., $X = C^{00} = g^{\mu\nu}\partial_\mu \Phi^0 \partial_\nu \Phi^0 \xrightarrow{\Phi^A = \alpha x^A} C^{00} = \alpha^2 g^{00}$
    - $\longrightarrow$ Potential for g^{00}

Criteria for degravitation

- **Existence of a Minkowski (degravitating) solution**: equations of motion must be solved by $g = \eta$ for arbitrary Λ
- **No fine-tuning**: Tune integration constants, not parameters, against Λ
- **Massless tensors**: Tensor mass generically is huge, $m \sim O(\Lambda^{1/2})$, unless they are exactly massless. (LIGO: $m < 10^{-22}$ eV)
- **No pathologies**: No ghosts, tachyons, gradient instabilities, infinite strong coupling, instantaneous modes
- **UV insensitivity**: Higher-derivative EFT corrections should not introduce new modes at low energy

Strategy

1. Identify parameter space of Lorentz-violating massive gravity which satisfies these criteria

2. Look for symmetries that protect our parameter choice

3. Determine fluid building blocks

4. Solve cosmological constant problem

Analysis

- Work in unitary gauge at linear level,

$$\Phi^A = (\alpha t, \beta x^i), \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Most general SO(3)-invariant mass term
Dubovsky hep-th/0409124

$$\mathscr{L}_{\text{mass}} = \frac{M_{\text{Pl}}^2}{2}\left(m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00} h_{ii}\right)$$

- Massless tensors: $m_2 = 0$

- Stability: $m_1 = 0$ (see our paper for gory details! 1805.05937)

Searching for a symmetry

- With our parameter choice in hand, $m_1 = m_2 = 0$, we need to find a symmetry to protect it
  - Otherwise we're just fine-tuning and not solving anything!
- Several candidate symmetries
  - Most either don't degravitate or are UV-sensitive
- Only symmetry that works: **time-dependent, volume-preserving spatial diffeomorphism**

Time-dependent, volume-preserving spatial diffeomorphisms

- Fluid language:

$$\Phi^i \to \Psi^i(\Phi^0, \Phi^i) \quad \text{with} \quad \det\left(\frac{\partial \Psi^i}{\partial \Phi^j}\right) = 1$$

- Massive gravity language: break diffs while leaving

$$x^i \to x^i + \xi^i(t, x^j) \quad \text{with} \quad \partial_i \xi^i = 0$$

- Closely related to time-*independent* volume-preserving spatial diffs, which set $m_2 = 0$ and forbid massive tensors

- Adding time dependence further restricts $m_1 = 0$, as necessary for stability

Building blocks of degravitation

- The building blocks invariant under our symmetry,

$$X = (\partial\Phi^0)^2$$

$$Yb = \frac{\det(\partial\Phi)}{\sqrt{-g}}$$

- Our degravitating theory is therefore

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 U(X, Yb)\right]$$

- Unique theory that satisfies our criteria

- This describes a finite-temperature superfluid!

Particle content

- Two massless tensors ($\omega^2 = p^2$)
- No vectors
- One *global* scalar ($\omega^2 = 0$)
  - Propagates due to higher-derivative EFT corrections
  - Sounds weird, but well-known from ghost condensation

Hints from the UV

- Including next-to-leading operators in derivative expansion, scalar dispersion relation becomes

$$\left(m_0^2 - \frac{m_4^4}{m_3^2}\right)\omega^2 = ap^4$$

- a: dimensionless constant determined by UV physics
- Ghost-free condition: $m_0^2 - \frac{m_4^4}{m_3^2} > 0$
- No gradient instability: $a > 0$
- UV insensitive: $\omega^2 \propto \frac{p^4}{\mathscr{M}^2}, \quad \mathscr{M} \sim \text{cutoff}$

Aside: relation to unimodular gravity

- Well-known attempt to solve cosmological constant problem: unimodular gravity

$$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = \frac{1}{M_{\rm Pl}^2} \left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right)$$

- Traceless version of Einstein equations: Λ reintroduced as integration constant
- This is one of a class of theories invariant under transverse diffeomorphisms (TDiff), corresponding to U(Yb)

Aside: relation to unimodular gravity

GR:	Unimodular:
$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2} R$	$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2} \left[R + m^2 U(Yb)\right]$
Ghost condensate:	**Our theory:**
$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2} \left[R + m^2 U(X)\right]$	$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2} \left[R + m^2 U(X, Yb)\right]$

Our theory is to unimodular as ghost condensate is to GR!

Same symmetry-breaking pattern

UV sensitivity of unimodular/TDiff gravity

- Aside: we find that TDiff theories are UV-sensitive
  - Not a good solution to the CC problem!
- TDiff: $m_0 = m_3 = m_4$
- Lowest order, no mode: $\left(m_0^2 - \frac{m_4^4}{m_3^2}\right)\omega^2 = 0 = 0 \times \omega^2$
- Next order in EFT: Lorentz invariance implies a massless mode, $\frac{ap^4 + b\omega^2 p^2 + c\omega^4}{\mathscr{M}^2} = 0 \propto \frac{(\omega^2 - p^2)^2}{\mathscr{M}^2}$

Degravitating solutions in practice

- Finally, we can see how this all works! Example:

$$U(X, Yb) = \frac{K_1}{2}(X+1)^2 + \frac{K_2}{2}(Yb)^2$$

Ghost condensate plus term quadratic in Yb

- Has degravitating solutions!

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi^0 = t, \quad \Phi^i = \left(-\frac{4\Lambda}{K_2 m^2}\right)^{1/6} x^i$$

- Can degravitate any positive Λ for $K_2 < 0$ and vice versa

- No ghost: $K_1 > 0$

Degravitating solutions in practice

- Another simple example:

$$U(X, Yb) = -X + \gamma XYb - \frac{\lambda}{2}(Yb)^2$$

- Degravitating solutions:

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi^0 = \sqrt{\frac{2\Lambda}{m^2} - \frac{\lambda}{2\gamma^2}}t, \quad \Phi^i = \left(\frac{2\gamma^2\Lambda}{m^2} - \frac{\lambda}{2}\right)^{-1/6} x^i$$

- Can degravitate any $\Lambda > \lambda m^2/4\gamma^2$

- Ghost-free: $\lambda > 0$

Future directions

- Cosmology! Embed this model in a *realistic scenario*
- Is the Minkowski solution an attractor?
- Hints we may need to (softly) break shift symmetry

Summary

- New method for self-tuning Λ by breaking Lorentz
  - Circumvent Weinberg
- Unique theory: finite-temperature superfluid
- Next step: see whether this cancellation can occur dynamically

Bonus slides

Weinberg's famous no-go theorem

- Weinberg (1988); see also Padilla review 1502.05296
- Assume some field ϕ "eats up" vacuum energy density for arbitrary Λ, i.e.,

$$T^{\phi}_{\mu\nu} = -T^{\Lambda}_{\mu\nu}$$

  - We'll consider a single scalar field, but proof extends to arbitrary spins/ number of fields
- Assume *Poincaré-invariant* vacua,

$$\phi = \text{const}, \qquad g_{\mu\nu} = \eta_{\mu\nu}$$

- For constant ϕ and $g_{\mu\nu}$, the field equations are

$$\left.\frac{\partial \mathscr{L}}{\partial \phi}\right|_{g,\phi=\text{const.}} = 0, \qquad \left.\frac{\partial \mathscr{L}}{\partial g_{\mu\nu}}\right|_{g,\phi=\text{const.}} = 0$$

- Scenario 1: equations of motion hold independently. Translation invariance:

$$\delta_M x^\mu = M^\mu{}_\nu x^\nu \quad \Longrightarrow \quad \delta_M g_{\mu\nu} = 2M_{(\mu\nu)}, \quad \delta_M \mathscr{L} = \mathscr{L}\,\mathrm{Tr}\,M$$

- General variation of L, assuming ϕ eom, is

$$\delta\mathscr{L} = \frac{\partial\mathscr{L}}{\partial g_{\mu\nu}}\delta g_{\mu\nu} + \frac{\partial\mathscr{L}}{\partial\phi}\delta\phi \quad \Longrightarrow \quad \mathscr{L} g^{\mu\nu} M_{\mu\nu} = 2\frac{\partial\mathscr{L}}{\partial g_{\mu\nu}} M_{\mu\nu}$$

- This implies (on-shell)

$$\frac{\partial\mathscr{L}}{\partial g_{\mu\nu}} = \frac{1}{2}\mathscr{L} g^{\mu\nu} \quad \Longrightarrow \quad \mathscr{L} = -\sqrt{-g}(V(\phi) + 2\Lambda) + \text{derivatives}$$

- The metric eom requires V(ϕ) = -2Λ at min; *fine-tuning*

- Scenario 2: equations of motion imply each other,

$$2g_{\mu\nu}\frac{\partial \mathscr{L}}{\partial g_{\mu\nu}} = f(\phi)\frac{\partial \mathscr{L}}{\partial \phi}$$

$$\equiv \frac{\partial \mathscr{L}}{\partial \tilde{\phi}}, \qquad \tilde{\phi} \equiv \int \frac{\mathrm{d}\phi}{f(\phi)}$$

- For constant fields, this implies scaling symmetry,

$$\delta_\epsilon g_{\mu\nu} = 2\epsilon g_{\mu\nu}, \quad \delta_\epsilon \tilde{\phi} = -\epsilon \quad \Longrightarrow \quad \delta_\epsilon \mathscr{L} = 0$$

- This requires

$$\mathscr{L} = -V_0\sqrt{-\hat{g}} + \text{derivatives}, \qquad \hat{g}_{\mu\nu} \equiv e^{2\tilde{\phi}} g_{\mu\nu}$$

implying (by $g_{\mu\nu}$ equation)

$$V_0 e^{4\tilde{\phi}} = 0$$

- Our equation of motion (dropping tildes),

$$V_0 e^{4\phi} = 0,$$

has two solutions:

  - $V_0 = 0$: *fine-tuning again*
  - $e^\phi = 0$: *all particles massless*

- QED