# Cosmology and a Massive Graviton

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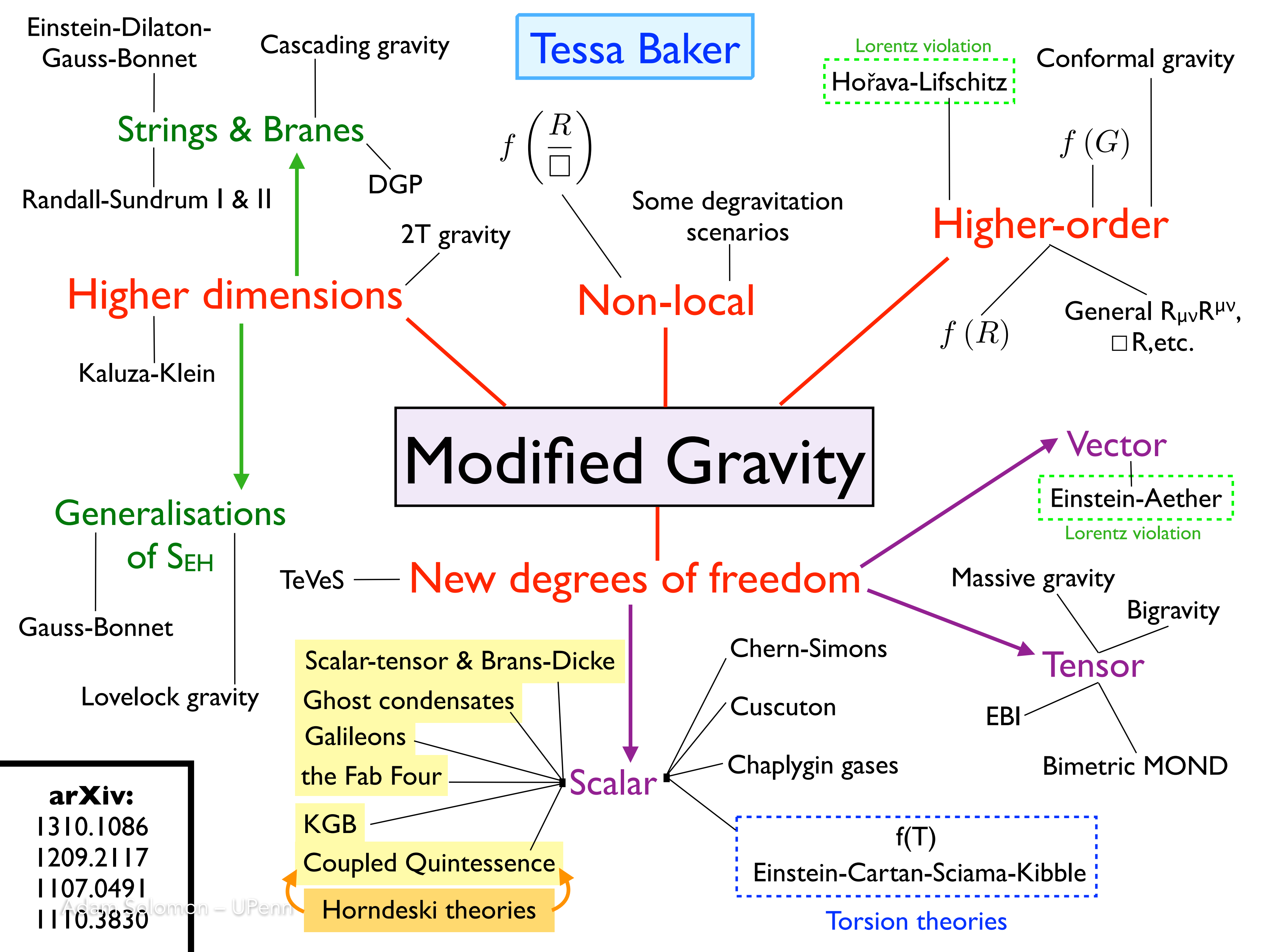
# The cosmological constant problem is hard

- Old CC problem: why isn't the CC enormous?
  - Vacuum energy induces large CC
  - Universe accelerates long before structure can form
- New CC problem: why isn't it zero?
  - The Universe is accelerating! Implies tiny but non-zero CC
  - Require *technical naturalness*, otherwise reintroduce old CC problem

# The CC problem is a problem of gravity

- Particle physics tells us the vacuum energy, and gravity translates that into cosmology
- The CC affects gravity at ultra-large distances, where we have few complementary probes of GR
- Does the CC problem point to IR modifications of GR?
- Can address the old problem, new problem, or both

# Modifying gravity is also hard



Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
DGP
Randall-Sundrum I & II
2T gravity
Kaluza-Klein
Generalisations of $S_{EH}$
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
New degrees of freedom
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
arXiv:
1310.1086
1209.2117
1107.0491
1110.3830

# Modifying gravity is also hard

- It is remarkably tricky to move away from GR in theory space
- Ghosts/instabilities
- Long-range fifth forces

# Massive gravity is a promising way forward

- Conceptual simplicity
  - GR: unique theory of a massless spin-2
  - Mathematical simplicity? Depends on your aesthetics
- Has potential to address both CC problems
  - Old CC problem: *degravitation*
    - Yukawa suppression lessens sensitivity of gravity to CC
    - Degravitation in LV massive gravity: work in progress (with Justin Khoury, Jeremy Sakstein)
- Technically natural small parameter
  - Graviton mass protected by broken diffs

# How to build a massive graviton
# Step 1: Go linear

- Consider a linearized metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{\rm Pl}} h_{\mu\nu}$$

The Einstein-Hilbert action at quadratic order is

$$\mathcal{L} = -\frac{1}{4} h^{\mu\nu} (\mathcal{E}h)_{\mu\nu} + \frac{1}{2M_{\rm Pl}} h_{\mu\nu} T^{\mu\nu}$$

with the Lichnerowicz operator defined by

$$(\mathcal{E}h)^{\mu\nu} = -\frac{1}{2} \varepsilon^{\mu\alpha\rho\cdot} \varepsilon^{\nu\beta\sigma}{}_{\cdot} \partial_\alpha \partial_\beta h_{\rho\sigma}$$

This action is invariant under linearized diffeomorphisms

$$h_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}$$

# How to build a massive graviton
# Step 1: Go linear

- Unique healthy (ghost-free) mass term (Fierz and Pauli, 1939):

$$\mathcal{L} = -\frac{1}{4}h^{\mu\nu}(\mathcal{E}h)_{\mu\nu} - \frac{1}{8}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) + \frac{1}{2M_{\rm Pl}}h_{\mu\nu}T^{\mu\nu}$$

- This massive graviton contains five polarizations:
  - 2 x tensor
  - 2 x vector
  - 1 x scalar
- Fierz-Pauli tuning: Any other coefficient between $h_{\mu\nu}h^{\mu\nu}$ and $h^2$ leads to a ghostly sixth degree of freedom

# dRGT Massive Gravity in a Nutshell

- The unique non-linear action for a single massive spin-2 graviton is

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R$$
$$+ m^2 M_g^2 \int d^4x \sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

  where $f_{\mu\nu}$ is a reference metric which must be chosen at the start

- $\beta_n$ are interaction parameters; the graviton mass is $\sim m^2\beta_n$
- The $e_n$ are elementary symmetric polynomials given by…

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R + m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

For a matrix X, the elementary symmetric polynomials are ([] = trace)

$$e_0(X) \equiv 1,$$
$$e_1(X) \equiv [X],$$
$$e_2(X) \equiv \frac{1}{2}\left([X]^2 - [X^2]\right),$$
$$e_3(X) \equiv \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right),$$
$$e_4(X) \equiv \det(X)$$

# An aesthetic aside

- The potentials in the last slide are ugly
- Lovely structure in terms of vielbeins and differential forms:

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b_\nu, \qquad f_{\mu\nu} = \eta_{ab} f^a{}_\mu f^b_\nu$$

$$e_1(X) \sim \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge f^d$$

$$e_2(X) \sim \varepsilon_{abcd} e^a \wedge e^b \wedge f^c \wedge f^d$$

$$e_3(X) \sim \varepsilon_{abcd} e^a \wedge f^b \wedge f^c \wedge f^d$$

# Much ado about a reference metric?

- There is a simple (heuristic) reason that massive gravity needs a second metric: you can't construct a non-trivial interaction term from one metric alone:

$$g^{\mu\alpha}g_{\nu\alpha} = \delta^\mu_\nu, \quad (g_{\mu\nu})^2 = 4, \quad \ldots$$

- We need to introduce a second metric to construct interaction terms.
- Can be Minkowski, (A)dS, FRW, etc., or even dynamical
- This points the way to a large family of theories with a massive graviton

# Theories of a massive graviton

Examples of this family of massive gravity theories:

- $g_{\mu\nu}$, $f_{\mu\nu}$ dynamical: bigravity (Hassan, Rosen: 1109.3515)
  - One massive graviton, one massless
- $g_{\mu\nu}$, $f_{1,\mu\nu}$, $f_{2,\mu\nu}$, …, $f_{n,\mu\nu}$, with various pairs coupling à la dRGT: multigravity (Hinterbichler, Rosen: 1203.5783)
  - n-1 massive gravitons, one massless
- Massive graviton coupled to a scalar (e.g., quasidilaton, mass-varying)

# The search for viable massive cosmologies

- **No stable FLRW solutions in dRGT massive gravity**
- Way out #1: large-scale inhomogeneites
- Way out #2: generalize dRGT
  - Break translation invariance (de Rham+: 1410.0960)
  - Generalize matter coupling (de Rham+: 1408.1678)
- Way out #3: new degrees of freedom
  - Scalar (mass-varying, f(R), quasidilaton, etc.)
  - Tensor (bi/multigravity) (Hassan/Rosen: 1109.3515)

# Massive bigravity has self-accelerating cosmologies

Consider FRW solutions

$$ds_g^2 = a^2\left(-d\tau^2 + d\vec{x}^2\right),$$
$$ds_f^2 = -X^2 d\tau^2 + Y^2 d\vec{x}^2$$

NB: g = physical metric (matter couples to it)

Bianchi identity fixes X

New dynamics are entirely controlled by y = Y/a

# Massive bigravity has self-accelerating cosmologies

The Friedmann equation for g is

$$3\mathcal{H}^2 = \frac{a^2\rho}{M_g^2} + m^2a^2\left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right)$$

The Friedmann equation for f becomes algebraic after applying the Bianchi constraint:

$$\beta_3 y^4 + (3\beta_2 - \beta_4)y^3 + 3(\beta_1 - \beta_3)y^2 + \left(\frac{\rho}{M_{\rm Pl}^2 m^2} + \beta_0 - 3\beta_2\right)y - \beta_1 = 0$$

# Massive bigravity has self-accelerating cosmologies

$$3\mathcal{H}^2 = \frac{a^2\rho}{M_g^2} + m^2a^2\left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right)$$

$$\beta_3 y^4 + (3\beta_2 - \beta_4)y^3 + 3(\beta_1 - \beta_3)y^2$$
$$+ \left(\frac{\rho}{M_{\rm Pl}^2 m^2} + \beta_0 - 3\beta_2\right) y - \beta_1 = 0$$

At late times, ρ → 0 and so y → const.

The mass term in the Friedmann equation approaches a constant – dynamical dark energy

# Massive bigravity vs. ΛCDM

Y. Akrami, T. Koivisto, and M. Sandstad [arXiv:1209.0457]
See also F. Könnig, A. Patil, and L. Amendola [arXiv:1312.3208];
ARS, Y. Akrami, and T. Koivisto [arXiv:1404.4061]

$\sim B_1 \mathrm{Tr}[\sqrt{g^{-1}f}]$ $\quad B_i = \frac{m^2}{H_0^2}\beta_i$

| Model | $B_0$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Omega_m$ | $\chi^2_{min}$ | p-value | log-evidence |
|---|---|---|---|---|---|---|---|---|---|
| ΛCDM | free | 0 | 0 | 0 | 0 | free | 546.54 | 0.8709 | -278.50 |
| $(B_1, \Omega_m^0)$ | 0 | free | 0 | 0 | 0 | free | 551.60 | 0.8355 | -281.73 |
| $(B_2, \Omega_m^0)$ | 0 | 0 | free | 0 | 0 | free | 894.00 | < 0.0001 | -450.25 |
| $(B_3, \Omega_m^0)$ | 0 | 0 | 0 | free | 0 | free | 1700.50 | < 0.0001 | -850.26 |
| $(B_1, B_2, \Omega_m^0)$ | 0 | free | free | 0 | 0 | free | 546.52 | 0.8646 | -279.77 |
| $(B_1, B_3, \Omega_m^0)$ | 0 | free | 0 | free | 0 | free | 542.82 | 0.8878 | -280.10 |
| $(B_2, B_3, \Omega_m^0)$ | 0 | 0 | free | free | 0 | free | 548.04 | 0.8543 | -280.91 |
| $(B_1, B_4, \Omega_m^0)$ | 0 | free | 0 | 0 | free | free | 548.86 | 0.8485 | -281.42 |
| $(B_2, B_4, \Omega_m^0)$ | 0 | 0 | free | 0 | free | free | 806.82 | < 0.0001 | -420.87 |
| $(B_3, B_4, \Omega_m^0)$ | 0 | 0 | 0 | free | free | free | 685.30 | 0.0023 | -351.14 |
| $(B_1, B_2, B_3, \Omega_m^0)$ | 0 | free | free | free | 0 | free | 546.50 | 0.8582 | -279.61 |
| $(B_1, B_2, B_4, \Omega_m^0)$ | 0 | free | free | 0 | free | free | 546.52 | 0.8581 | -279.56 |
| $(B_1, B_3, B_4, \Omega_m^0)$ | 0 | free | 0 | free | free | free | 546.78 | 0.8563 | -280.00 |
| $(B_2, B_3, B_4, \Omega_m^0)$ | 0 | 0 | free | free | free | free | 549.68 | 0.8353 | -282.89 |
| $(B_1, B_2, B_3, B_4, \Omega_m^0)$ | 0 | free | free | free | free | free | 546.50 | 0.8515 | -279.60 |
| full bigravity model | free | free | free | free | free | free | 546.50 | 0.8445 | -279.82 |

# Beyond the background

Cosmological perturbation theory in massive bigravity is a huge cottage industry and the source of many PhD degrees. See:

Cristosomi, Comelli, and Pilo, 1202.1986
**ARS**, Akrami, and Koivisto, 1404.4061
Könnig, Akrami, Amendola, Motta, and **ARS**, 1407.4331
Könnig and Amendola, 1402.1988
Lagos and Ferreira, 1410.0207
Cusin, Durrer, Guarato, and Motta, 1412.5979

and many more for more general matter couplings!

# Scalar perturbations in massive bigravity

- Our approach (1407.4331 and 1404.4061):
- Linearize metrics around FRW backgrounds, restrict to scalar perturbations {$E_{g,f}$, $A_{g,f}$, $F_{g,f}$, and $B_{g,f}$}:

$$ds_g^2 = a^2 \left\{-(1+E_g)d\tau^2 + 2\partial_i F_g d\tau dx^i + \left[(1+A_g)\delta_{ij} + \partial_i\partial_j B_g\right] dx^i dx^j\right\}$$
$$ds_f^2 = -X^2(1+E_f)d\tau^2 + 2XY\partial_i F_f d\tau dx^i + Y^2\left[(1+A_f)\delta_{ij} + \partial_i\partial_j B_f\right] dx^i dx^j$$

- Full linearized Einstein equations (in cosmic or conformal time) can be found in **ARS**, Akrami, and Koivisto, arXiv:1404.4061

# Scalar fluctuations can suffer from instabilities

- Usual story: solve perturbed Einstein equations in subhorizon, quasistatic limit: $k^2\Phi \gg H^2\Phi \sim H\dot{\Phi} \sim \ddot{\Phi}$
- This is valid only if perturbations vary on Hubble timescales
- Cannot trust quasistatic limit if perturbations are unstable
- Check for instability by solving full system of perturbation equations

# Scalar fluctuations can suffer from instabilities

- Degree of freedom count: ten total variables
  - Four $g_{\mu\nu}$ perturbations: $E_g$, $A_g$, $B_g$, $F_g$
  - Four $f_{\mu\nu}$ perturbations: $E_f$, $A_f$, $B_f$, $F_f$
  - One perfect fluid perturbation: $\chi$
- Eight are redundant:
  - Four of these are nondynamical/auxiliary ($E_g$, $F_g$, $E_f$, $F_f$)
  - Two can be gauged away
  - After integrating out auxiliary variables, one of the dynamical variables becomes auxiliary – related to absence of ghost!
- End result: only two independent degrees of freedom
- NB: This story is deeply indebted to Lagos and Ferreira

# Scalar fluctuations can suffer from instabilities

- Choose g-metric Bardeen variables:

$$\Phi \equiv A_g - H\left(F_g + B_g'\right)$$

$$\Psi \equiv E_g - H\left(F_g + B_g'\right) - F_g' - B_g''$$

- Then *entire* system of 10 perturbed Einstein/fluid equations can be reduced to two coupled equations:

$$X_i'' + F_{ij}X_j' + S_{ij}X_j = 0$$

where

$$X_i = \{\Phi, \Psi\}$$

# Scalar fluctuations can suffer from instabilities

- Ten perturbed Einstein/fluid equations can be reduced to two coupled equations:

$$X_i'' + F_{ij}X_j' + S_{ij}X_j = 0$$

where

$$X_i = \{\Phi, \Psi\}$$

- Under assumption (WKB) that $F_{ij}$, $S_{ij}$ vary slowly, this is solved by

$$X_i = X_i^0 e^{i\omega N}$$

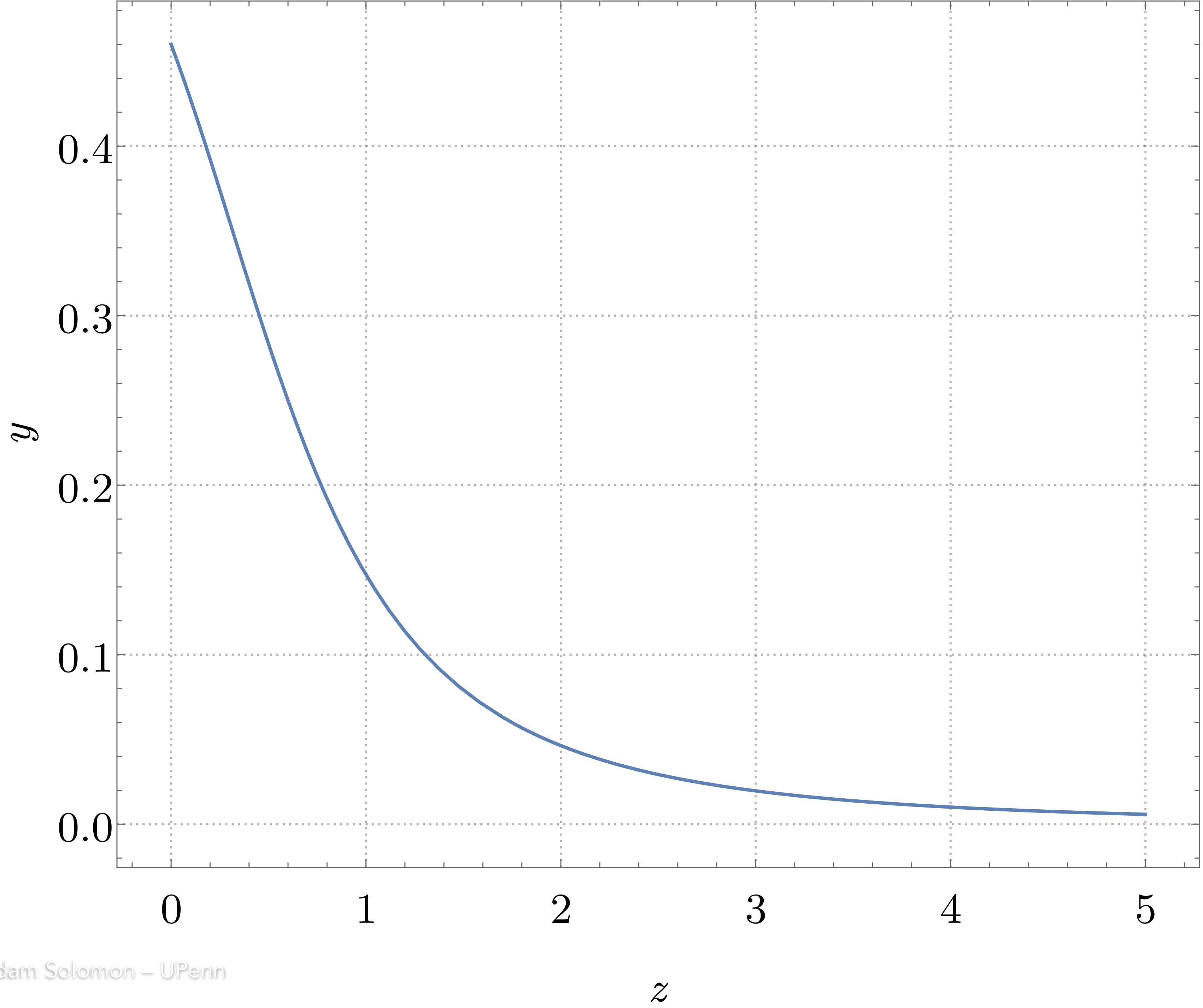
with N = ln a

# Scalar fluctuations can suffer from instabilities

- $B_1$-only model – simplest allowed by background

$$\omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2}$$

- Unstable for small y (early times)
- NB: Gradient instability


0.4
0.3
0.2
0.1
0.0
y
0
1
2
3
4
5
z

# Scalar fluctuations can suffer from instabilities

- $B_1$-only model – simplest allowed by background

$$\omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2}$$

- Unstable for small y (early times)
- For realistic parameters, model is only (linearly) stable for z <~ 0.5

# Instability does not rule models out

- Instability → breakdown of linear perturbation theory
  - Nothing more
  - Nothing less
- Cannot take quasistatic limit
- Need nonlinear techniques to make structure formation predictions

# Do nonlinearities save us?

- Helicity-0 mode of the massive graviton has nonlinear derivative terms (cf. galileons)
- Linear perturbation theory misses these
- Some evidence that the gradient instability is cured by these nonlinearities! (Aoki, Maeda, Namba 1506.04543)
  - Morally related to the Vainshtein mechanism
- How can we study nonlinear perturbations in detail?

# Strategy for "quasilinear" perturbations

- Bigravity's degrees of freedom:
  - 1 massive graviton (two helicity-2, two helicity-1, one helicity-0)
  - 1 massless graviton (two helicity-2)
- Goal: keep nonlinearities in spin-0 mode while leaving other fluctuations linear

# Cosmology as a perturbation of flat space

- Helicity decomposition only well-defined around flat background
- Q: How can we isolate helicity-0 mode in cosmological perturbations?
- A: Use Fermi normal coordinates, for which

$$g_{\mu\nu}^{\mathrm{FRW}}(x) = \eta_{\mu\nu} + \mathcal{O}(H^2\vec{x}^2)$$

# Cosmology as a perturbation of flat space

- Starting from the FRW metric in comoving coordinates,

$$\mathrm{d}s^2 = -\mathrm{d}t_\mathrm{c}^2 + a^2(t_\mathrm{c})\mathrm{d}\vec{x}_\mathrm{c}^2$$

the coordinate change

$$t_\mathrm{c} = t - \frac{1}{2}H\vec{x}^2$$

$$\vec{x}_\mathrm{c} = \frac{\vec{x}}{a(t)}\left(1 + \frac{1}{4}H^2\vec{x}^2\right)$$

yields

$$\mathrm{d}s^2 = -\left[1 - (\dot{H} + H^2)\vec{x}^2\right]\mathrm{d}t^2 + \left(1 - \frac{1}{2}H^2\vec{x}^2\right)\mathrm{d}\vec{x}^2 + \mathcal{O}(H^4\vec{x}^4)$$

# Cosmology as a perturbation of flat space

- So within the horizon we can write FRW as Minkowski space plus a small perturbation,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$
$$h_{\mu\nu} = \mathrm{diag}\left(\dot{H} + H^2, -\frac{1}{2}H^2\delta_{ij}\right)\vec{x}^2$$

# Bimetric Fermi normal coordinates

- In bigravity with two FRW metrics,

$$\mathrm{d}s_g^2 = -\mathrm{d}t_\mathrm{c}^2 + a^2\mathrm{d}\vec{x}_\mathrm{c}^2$$
$$\mathrm{d}s_f^2 = -X^2\mathrm{d}t_\mathrm{c}^2 + Y^2\mathrm{d}\vec{x}_\mathrm{c}^2$$

  we clearly need two different Fermi coordinates.

- Fermi coordinates $\Phi^a$ for f metric. Method: start with f-metric comoving coordinates $\Phi_\mathrm{c}{}^a$ such that

$$\mathrm{d}s_f^2 = -\mathrm{d}(\Phi_\mathrm{c}^0)^2 + Y^2\mathrm{d}\vec{x}_\mathrm{c}^2$$

  and then build FNC $\Phi^a$ from $\Phi_\mathrm{c}{}^a$ as in previous slide

- We identify $\Phi^a$ as the Stückelberg fields associated to broken diff invariance

# Bimetric Fermi normal coordinates

- The FNC for the bimetric FRW backgrounds are related by the helicity-0 Stückelberg,

$$\Phi^a = x^a + \partial^a \pi$$

$$\pi = \iint (1 - X)\, \mathrm{d}t^2 + \frac{1}{2}(y - 1)\vec{x}^2$$

# To the decoupling limit

- This is all suggestive of the decoupling limit of bigravity:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{1}{M_{\rm Pl}} h_{\mu\nu}$$

$$f_{ab}(\Phi) = \eta_{ab} + \frac{1}{M_f} v_{ab}$$

$$\Phi^a = x^a + \Lambda_3^{-3} \partial^a \pi$$

with $\Lambda_3{}^3 = m^2 M_{Pl}$, and take the scaling limit

$$M_{\rm Pl}, M_f \to \infty, \qquad m \to 0$$

while keeping $\Lambda_3$, $M_{Pl}/M_f$, and $\beta_n$ fixed

- DL leaves leading interactions between helicity modes

# The decoupling limit

- The action in the DL is

$$\mathcal{L} = -\frac{1}{4}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{4}v^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}v_{\alpha\beta} + \frac{\Lambda_3^3}{2}\left(h_{\mu\nu}X^{\mu\nu} + \frac{M_{\rm Pl}}{M_f}v_{\mu\nu}\tilde{Y}^{\mu\nu}\right)$$

The first two terms are linearized Einstein-Hilbert, and

$$X^{\mu\nu} = -\frac{1}{2}\sum_{n=1}^{3}\frac{\beta_n}{(3-n)!n!}\varepsilon^{\mu\cdots}\varepsilon^{\nu\cdots}(\eta+\Pi)^n\eta^{3-n}$$

$$\tilde{Y}^{\mu\nu} = -\frac{1}{2}\sum_{n=1}^{3}\frac{\beta_n}{(4-n)!(n-1)!}\varepsilon^{\mu\cdots}\varepsilon^{\nu\cdots}(\eta+\Sigma)^{4-n}\eta^{n-1}$$

where $\Pi_{\mu\nu} \equiv \Lambda_3^{-3}\partial_\mu\partial_\nu\pi, \qquad \Sigma_{\mu\nu} \equiv \Lambda_3^{-3}\partial_\mu\partial_\nu\rho$

$$\frac{\partial\pi(x)}{\partial x^\mu} = -\frac{\partial\rho(\Phi)}{\partial\Phi^\mu}$$

# Cosmological perturbations in the decoupling limit

- The DL equations of motion yield correct background cosmological equations ✓
- Nonlinear subhorizon structure formation by perturbing in DL

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \chi_{\mu\nu}$$

$$v_{ab} = \bar{v}_{ab} + w_{ab}$$

$$\pi = \bar{\pi} + \phi$$

$$\rho = \bar{\rho} + \psi$$

- Can consistently keep $\chi_{\mu\nu}$ and $w_{ab}$ linear while retaining nonlinearities in $\varphi$ and $\psi$!

# Example: $\beta_2$-only model

- The action in the DL is

$$\mathcal{L} = -\frac{1}{4}h\mathcal{E}h - \frac{1}{4}v\mathcal{E}v - \frac{\Lambda_3^3}{8}\beta_2\varepsilon\varepsilon\left(h(\eta+\Pi)^2\eta + v(\eta+\Sigma)^2\eta\right)$$

- Perturb to second order:

$$\mathcal{L}_2 = -\frac{1}{4}\chi\mathcal{E}\chi - \frac{1}{4}w\mathcal{E}w - \frac{\Lambda_3^3}{8}\beta_2\varepsilon\varepsilon\left(\bar{h}\Phi^2\eta + 2\chi\Phi(\eta+\bar{\Pi})\eta + \bar{v}\Psi^2\eta + 2w\Psi(\eta+\bar{\Sigma})\eta\right)$$

Can be fully diagonalized:

$$\chi_{\mu\nu} = \hat{\chi}_{\mu\nu} + \beta_2\phi(\eta+\bar{\Pi})_{\mu\nu}$$
$$w_{\mu\nu} = \hat{w}_{\mu\nu} + \beta_2\psi(\eta+\bar{\Sigma})_{\mu\nu}$$

# Example: $\beta_2$-only model

- This leaves us with (after int by parts, removing dual galileon)

$$\mathcal{L}_2 = -\frac{1}{4}\hat{\chi}\mathcal{E}\hat{\chi} - \frac{1}{4}\hat{w}\mathcal{E}\hat{w} + c_{\mathrm{k}}\dot{\phi}^2 - c_{\mathrm{g}}\nabla^2\phi$$

where the coefficients of the kinetic and gradient terms are

$$c_{\mathrm{k}} = \frac{6}{X}\left[\beta_2 y(1+Xy) - H^2(X+y)\right]$$

$$c_{\mathrm{g}} = \frac{2}{y}\left[\beta_2\left(X+2y+2Xy^2+y^3\right) - (X+5y)H^2 - 4\dot{H}y\right]$$

$c_k>0$ is equivalent to the Higuchi bound: useful consistency check! NB $\beta_2$ is not a linearly unstable model

# Summary

- Massive gravity is a promising approach to the cosmological constant problems
- Bimetric massive gravity has cosmological backgrounds competitive with $\Lambda$CDM
- These models have linear instabilities
- A deeper look into quasilinear behavior of perturbations can shed light on endpoint of instability
- Resuscitate massive (bi)gravity as a target for observations?