

Cooling Processes : Main Points

Goldsmith & Langer 1978

- ① For low $n(H_2)$, $\Lambda(A)/n(A)$ $\propto n(H_2)$, is independent of $X(A) / (dv/dr)$ escape prob.
- optically thin limit, $\beta = 1$
- $$\Lambda(A) \propto X(A) n^2(H_2)$$
- ② for optically thin case,
 $\Lambda(A) \propto n(H_2)$ long after lower transitions are thermalized
ie get little information on cooling by CO from $J=1-0$ line
if $n > 100$

Goldsmith & Lange

No. 3, 1978

DENSE INTERSTELLAR CLOUDS

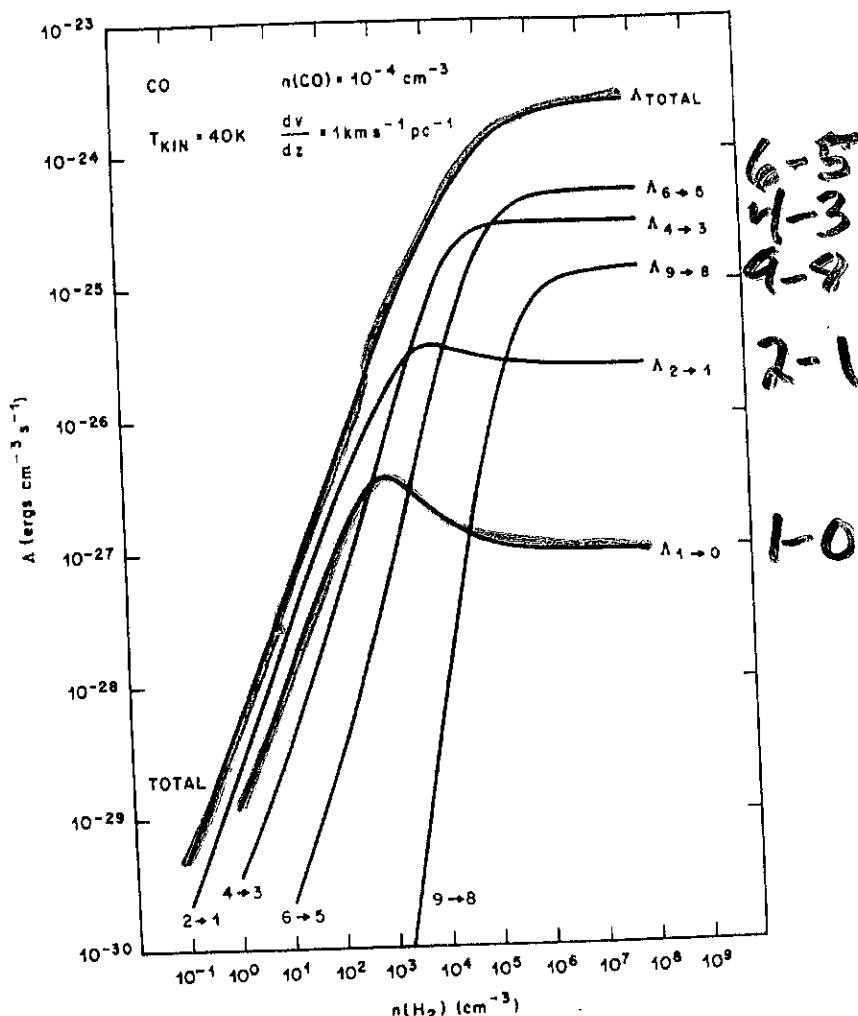


FIG. 1.—Contributions from different transitions to CO cooling at a kinetic temperature of 40 K. The density of CO is chosen to make all transitions optically thin, and is held constant throughout. Note that the total cooling continues to be proportional to $n(\text{H}_2)$ long after $\Lambda_{1 \rightarrow 0}$ (which is proportional to $T_{A_{1 \rightarrow 0}}$) has become essentially independent of $n(\text{H}_2)$.

most cases (including CO and HCN) this is indicative of the optically thin limit in which the escape probability $\beta \rightarrow 1$ for all transitions. In this limit $\Lambda(A) \propto X(A)n^2(\text{H}_2)$; this is the regime characteristic of diffuse clouds and is also discussed by de Jong, Chu, and Dalgarno (1975) for carbon monoxide. This behavior of $\Lambda(A)$ can also occur when $\beta \ll 1$ if the collisional de-excitation rate C is much less than the spontaneous decay rate $A_{J \rightarrow I}$, as is the case for H_2O . As discussed by Linke *et al.* (1977), in this regime the flux from any individual transition will still be proportional to the molecular abundance.

transitions become optically thick when the maximum hydrogen density reaches 10^7 cm^{-3} .

b) Optically Thick

For thermalized transitions, the optically thick limit reduces the cooling per molecule. The cooling rates of the Figures (2)–(6), for large β , are proportional to $X(A)/(dv/dr)$; thus the cooling rate is independent of $X(A)$ and proportional to (dv/dr) .

Once the opacity in a particular transition becomes optically thick, the cooling must be that of a black body. The size and molecular density dependence can be seen to follow from the fact that for $\tau \gg 1$, $\beta \rightarrow \tau^{-1}$, and for a single transition, neglecting, for simplicity, the finite width of the emission line, the cooling rate per unit volume is

$$\Lambda(A)_{IJ} = \frac{A_{J \rightarrow I} h v_{IJ}}{\tau}$$

where f_J is the fractional population of level J . Using equation (4), we can write

$$\tau_{IJ} = \frac{h[f_J n(A)]}{dv/dr} B_{J \rightarrow I} [\text{ex}]$$

where $T_* \equiv h v_{IJ}/k$ and T_{EX} is the excitation temperature of the transition. The usual relations between the B 's and A 's (cf. Goldsmith & Langer 1974) give

$$\Lambda(A)_{IJ} = \frac{8\pi h v_{IJ}^4}{c^3} [\exp(T_*) - 1]$$

which has the form of the cooling rate for a single transition from a black body. The cooling rate for the I level is

For intermediate hydrogen densities, the hydrogen atoms thermalize all of the cooling rates. At higher opacities, the radiation trapping effect becomes dominant on the level populations. This leads to a situation in which the broken line in Figure 1 would be produced if the escape probability (eq. [2]) were proportional to the product of the level populations n_I and n_J with the cooling rate $\Lambda(A)_{IJ}$.

③ For optically thick & thermalized transitions, increasing opacity reduces cooling per molecule

$$\Lambda(A)/n(A) \propto (X(A)(dv/dr))^{-1}$$

Why?

$\tau \gg 1 \rightarrow$ cooling is blackbody

$$\rightarrow \beta \rightarrow \tau^{-1}$$

$$\Lambda(A)_{IJ} = \frac{A_{J \rightarrow I} h \gamma_{IJ} [f_J n(A)]}{\tau_{IJ}}$$

$$\text{but } \tau_{IJ} = \frac{h [f_J n(A)]}{dv/dr} B_{J \rightarrow I} \left(e^{\frac{hv}{kT_{ex}}} - 1 \right)$$

$$\rightarrow \Lambda(A)_{IJ} = \frac{8\pi h \gamma_{IJ}^4}{c^3} \left(e^{\frac{hv}{kT_{ex}}} - 1 \right)^{-1} \frac{dv}{dr}$$

$$\therefore \frac{\Lambda(A)}{n(A)} \propto \left(\frac{X(A)}{dv/dr} \right)^{-1} n(H_2)^{-1}$$

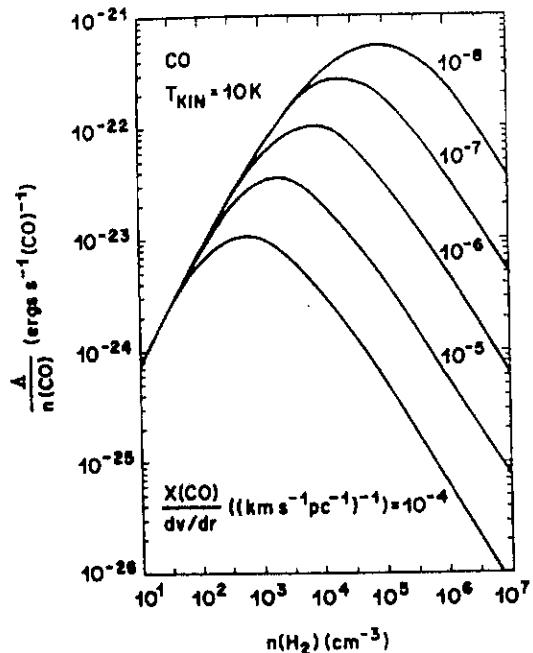


FIG. 2a

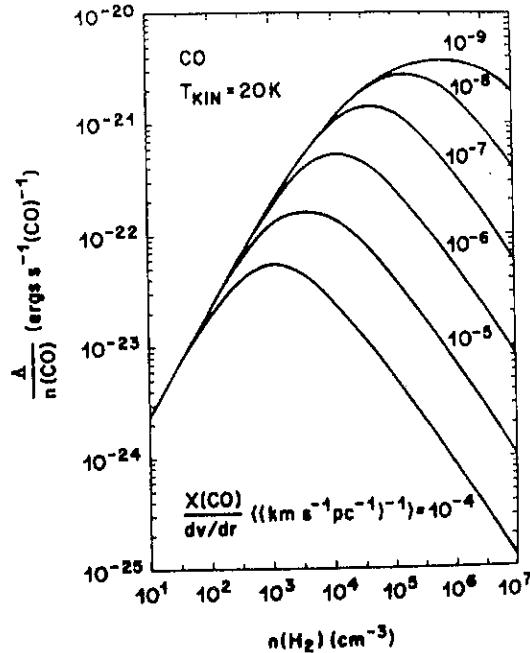


FIG. 2b

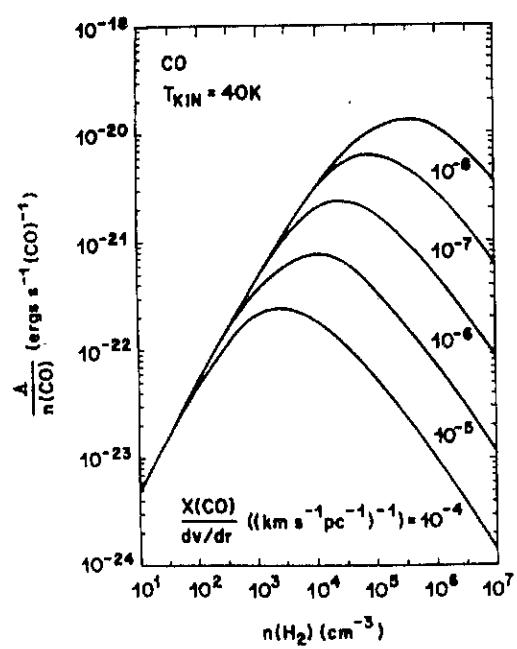


FIG. 2c

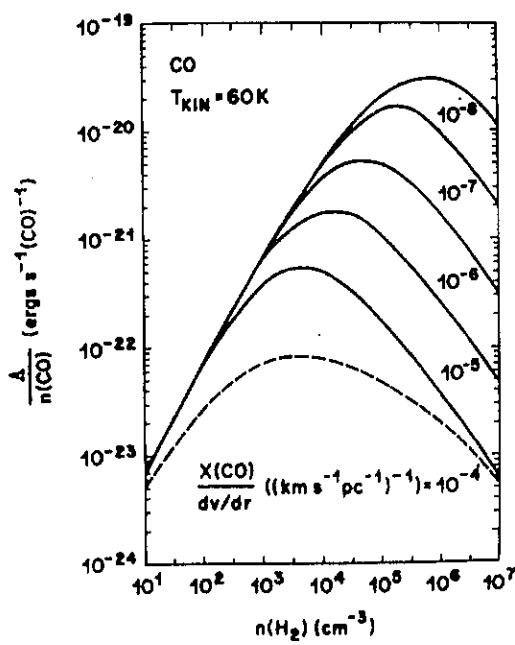


FIG. 2d

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2(a)-2(d).—Cooling per CO molecule as a function of $n(\text{H}_2)$ and CO fractional abundance for kinetic temperatures 10 K, 20 K, 40 K, and 60 K. The broken line in Fig. 2d indicates the cooling for $X(\text{CO})/(dv/dr) = 10^{-4} (\text{km s}^{-1} \text{pc}^{-1})^{-1}$ if the effect of trapping on the escape probabilities but not on the level populations is taken into account. The trapping reduces the population of levels $J \leq 3$ and increases the population of levels $J \geq 4$.

lly thick at any temperature ≥ 10 K. The peak cooling curve as a function of J thus occurs at a value J^* . Empirically we find that the

10 K, 20 K, 40 K, and 60 K in Figures 2a-2d. Each figure contains a number of curves parametrized by $X(\text{CO})/(dv/dr)$ with dv/dr in units of $\text{km s}^{-1} \text{pc}^{-1}$.

Important molecules

① CO, ^{13}CO , C^{18}O

- radiative trapping effects CO, so isotopes can be important at high densities

② CI

- fine-structure level at 23.4 K
- important at low (10-20K) temperatures

③ O_2

- efficiency comparable to CO
- rotational transitions

④ H_2O

- more important at high temperatures, densities

Total cooling dominated by
 ^{12}CO for $n < 10^3$

$^{13}\text{CO}, \text{C}^{18}\text{O}, \text{CI}, \text{O}_2$ 30-70% for $n > 10^3$

1978

for CO cooling alone; with the inclusion of various molecular species with greatly differing A -coefficients, however, the total cooling is dependent upon the molecular hydrogen density throughout the range of conditions considered here.

We next shall discuss some of the heating sources that may play a role in determining the thermal balance of dense clouds. The heating rate for each will be calculated and the conditions under which it might contribute significantly to the cloud heating analyzed.

a) Heating Mechanisms

The cooling rates that we have derived can be used to determine the time scale for thermal cooling in interstellar clouds. This time scale is estimated, from the formula $\tau_{\text{th}} = 1.5kT[\Lambda/n(\text{H}_2)]$ and from the rates in Table 4, to be less than 10^5 years over the range of densities 10^3 – 10^4 cm^{-3} for $T_{\text{kin}} > 8 \text{ K}$ —roughly an order of magnitude less than the free-fall time. This relatively short thermal time scale strongly suggests that there must be a heating source for these objects which maintains them in thermal equilibrium. Several possible sources of heating, which will be discussed below, are cosmic rays, H_2 formation, and gravitational contraction.

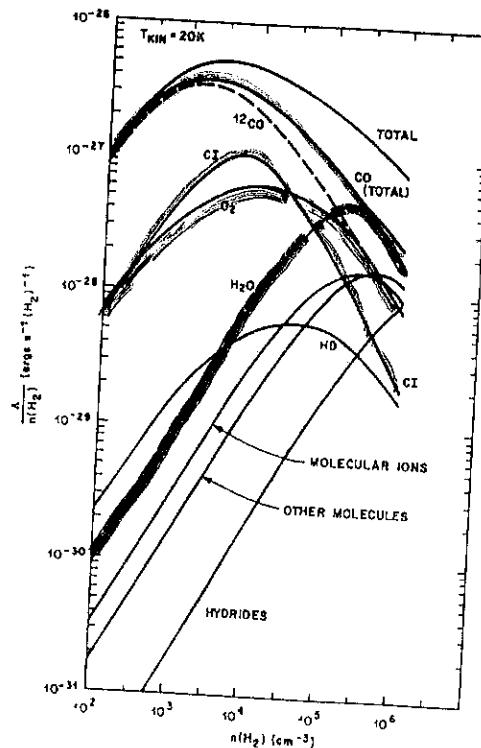


FIG. 7b

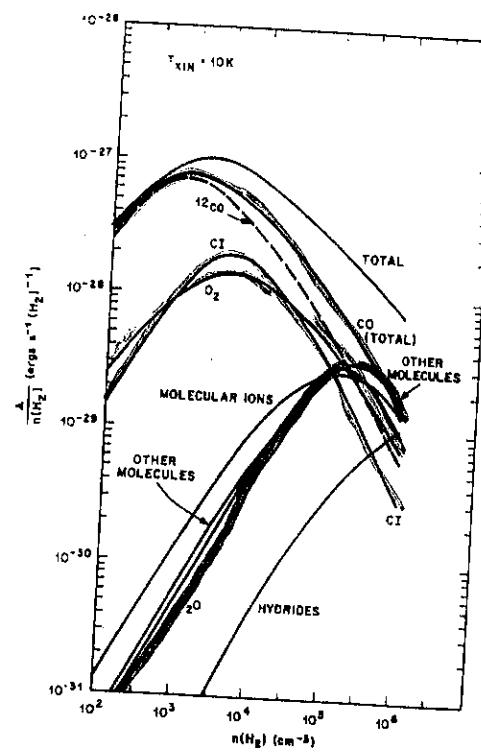


FIG. 7a

— CO
— CI

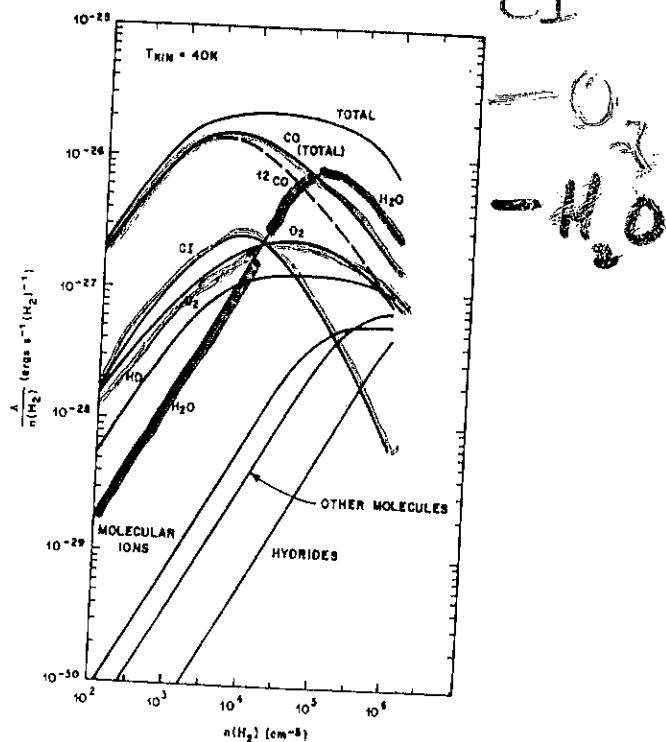


FIG. 7c

— CO
— CI
— H₂O

FIGS. 7(a)–7(c).—Total cooling per H_2 molecule as a function of H_2 density for kinetic temperatures 10 K, 20 K, and 40 K. The fractional abundance for each species has been taken from Table 3.

Thermal Balance

Cooling timescale $T_{th} = 1.5 kT [1/n(H_2)] \leq 10^5 \text{ yr}$

Heating sources

- cosmic rays (ionize H₂)
- H₂ formation (binding energy)
- gravitational contraction
- ambipolar diffusion (depends on Xe, B)

Clouds with internal heat sources

- T > 40 K → dust heats gas
(need $n \geq 10^5$ for strong coupling)
- 15 < T < 40 K (moderate luminosity)
→ need H₂ formation + ambipolar diffusion

Dark clouds (6-15 K, $10^2 - 10^4 \text{ cm}^{-3}$)

- cosmic rays can work (and so can the other 3 mechanisms under "other conditions")

How do we get physical quantities from molecular line observations?

ie T temperature

n density

N column density

τ optical depth

$\frac{[X]}{[H_2]}$ relative abundance

Two very common approximations or models are

(1) LTE "Local Thermodynamic Equilibrium"

(2) LVG "Large Velocity Gradient"

(1) LTE

We assume that the level populations of a molecule are given by a single excitation temperature, and that

$$T_{\text{ex}} = T_{\text{kin}} \quad (\text{kinetic temperature})$$

$$\text{so } \frac{N_e}{N_u} = \frac{g_e}{g_u} e^{\frac{hv}{kT_{\text{ex}}}}$$

recall radiative transfer equation

$$\Delta T_A = T_B - T_{\text{bg}} = \frac{hv}{k} \left[\frac{1}{e^{\frac{hv}{kT_{\text{ex}}}} - 1} - \frac{1}{e^{\frac{hv}{kT_{\text{bg}}}} - 1} \right] \times (1 - e^{-\tau})$$

$\tau \gg 1$, neglect T_{bg}

$$\rightarrow T_A = \frac{\frac{hv}{k}}{e^{\frac{hv}{kT_{\text{ex}}}} - 1}$$

↳ determine $T_{\text{ex}} = T_{\text{ex}}$ from observed T_A

$\tau \ll 1$, neglect T_{bg}

$$T_A = \tau \frac{h\nu}{e^{h\nu/kT_{\text{ex}}} - 1}$$

now $\tau = h\nu\phi_r [B_{\text{lu}}N_e - B_{\text{ul}}N_u]/c$

↳ line profile function

$$\int \phi_r dr = 1$$

$$\rightarrow T_A = \frac{hc^2\phi_r}{8\pi k\nu} A_{\text{ul}} N_u$$

$$\int T_A dV = \text{const} \times N_u$$

→ determine N_u from line integral

→ τ from N_u

→ T_{ex} from observed T_A , τ
($= T_{\text{min}}$)

Problem : LTE often is not a good approximation

(2) LVG - this model is basically the Sobolev approximation (developed for stellar winds) applied to molecular clouds

Main assumption is there are velocity gradients or other motions in the cloud, so that we can treat radiative transfer as a local problem.

(1) Statistical equilibrium

$$n_i(r) \sum_j P_{ij}(r) = \sum_j n_j(r) P_{ji}(r)$$

$$P_{ij} = \begin{cases} A_{ij} + B_{ij} \langle J_{ij}(r) \rangle + C_{ij}(r) & i \neq j \\ B_{ij} \langle J_{ij}(r) \rangle + C_{ij}(r) & i = j \end{cases}$$

$\langle J_{ij}(r) \rangle$ mean integrated radiation field at necessary frequency

(2) Radiative transfer

$$I = S(1 - e^{-\tau}) + I(0)e^{-\tau}$$

$$S = j/\kappa \quad \text{source function}$$

$$\tau = \int \kappa dz$$

$$\langle J_{ij}(r) \rangle = \int \frac{ds}{4\pi} \int_0^{\infty} I \phi(r') dr'$$
$$= (1 - \beta) S_{ij} + \beta B_r(\gamma_{ij}, T_{\text{b}})$$

$$\beta = \int \frac{ds}{4\pi} \int_0^{\infty} \phi(r') e^{-\tau} dr'$$

= escape probability of line

need to know $\kappa, j \rightarrow \tau$
 $\rightarrow \langle J_{ij} \rangle$

solve (1) & (2) iteratively to
get level populations

then get T_{ex} from

$$\frac{n_u}{n_e} = \frac{g_u}{g_e} e^{-\frac{h\nu}{kT_{\text{ex}}}}$$

then get T_A from (ie Rayleigh-Jeans limit)

$$T_A = T_{\text{ex}}(1 - e^{-z})$$

→ compare to observed line strengths

→ vary n , N/dv , T_{ex} until fit

$$\frac{\text{column density}}{\text{velocity width}}$$

→ need several different rotational transitions, maybe in different isotopes, to get well-constrained solution

→ assumes gas has uniform density and temperature

Determining kinetic temperature

(1) $T_K = T_B$ of optically thick, low J

^{12}CO lines

→ 10-20 K if no high-luminosity heat sources

→ $T > 40\text{K}$ in substantial regions of
+ few clouds w. large heating rates

problems - filling factors, temperature
gradients, embedded heat sources,
motions → what gas being measured

(2) Level populations of NH_3

→ agree fairly well w. CO

problems - optical depth, low angular
resolution + sensitivity, only
trace dense gas

(3) Level populations of rotational lines
of CO, CS, etc → very sensitive if
 $E_{\text{ul}} > kT_{\text{kin}}$
→ also get density information
problems - complicated modelling required,
observations at different λ

Determining Density

- (1) average from $N^{\frac{1}{3}}$ size or $M^{\frac{1}{3}}$ size
 $n \sim 5 - 10 \times 10^3 \text{ cm}^{-3}$
- clumpiness \rightarrow underestimates density
- (2) intensities of 2 lines which are not thermalized
- intensity a function of collision rate
- need to know collision rates \rightarrow only available for a few common molecules
 CO, CS, HCN
- particular transitions sensitive to small density range
ie $\text{CO } 1-0 - 2-1 \rightarrow 10^{2.5} < n < 10^4 \text{ cm}^{-3}$
- (3) use molecules with high n_{crit}
ie CS, HCN
- crude tracer of high- n gas
Cloud structure \rightarrow high density clumps in lower density interclump medium

(4) Multi-transition work (basically extension of (2))

- get density and temperature
- very sensitive to n if lines from subthermal regime included, especially if optically thin

problems - complicated models

- collisions rates mostly unknown
- observations at different λ

Determining Column Density

- (1) rare isotopes of CO
- optically thin + ubiquitous
 - not easy to put $> 10\%$ of C in CO
 - also not easy to deplete it
 - low transitions easily excited



↳ can be optically thick in large clouds

(2) other molecules

- abundance variations make them useless for total (H) column density
- if assume LTE, can get total N for that species
- or use many transitions
- then can compare this to $N(\text{H}_2)$ to get absolute abundance

$$\text{eg. } \frac{N(X)}{N(\text{H}_2)} = \frac{N(X)}{N(\text{CO})} \frac{N(\text{CO})}{A_v} \frac{A_v}{N(\text{H}_2)}$$

Heyer, Carpenter, & Snell
2001, ApJ, 551, 852

- outer galaxy survey
- 50" resolution + sampling
- ^{12}CO

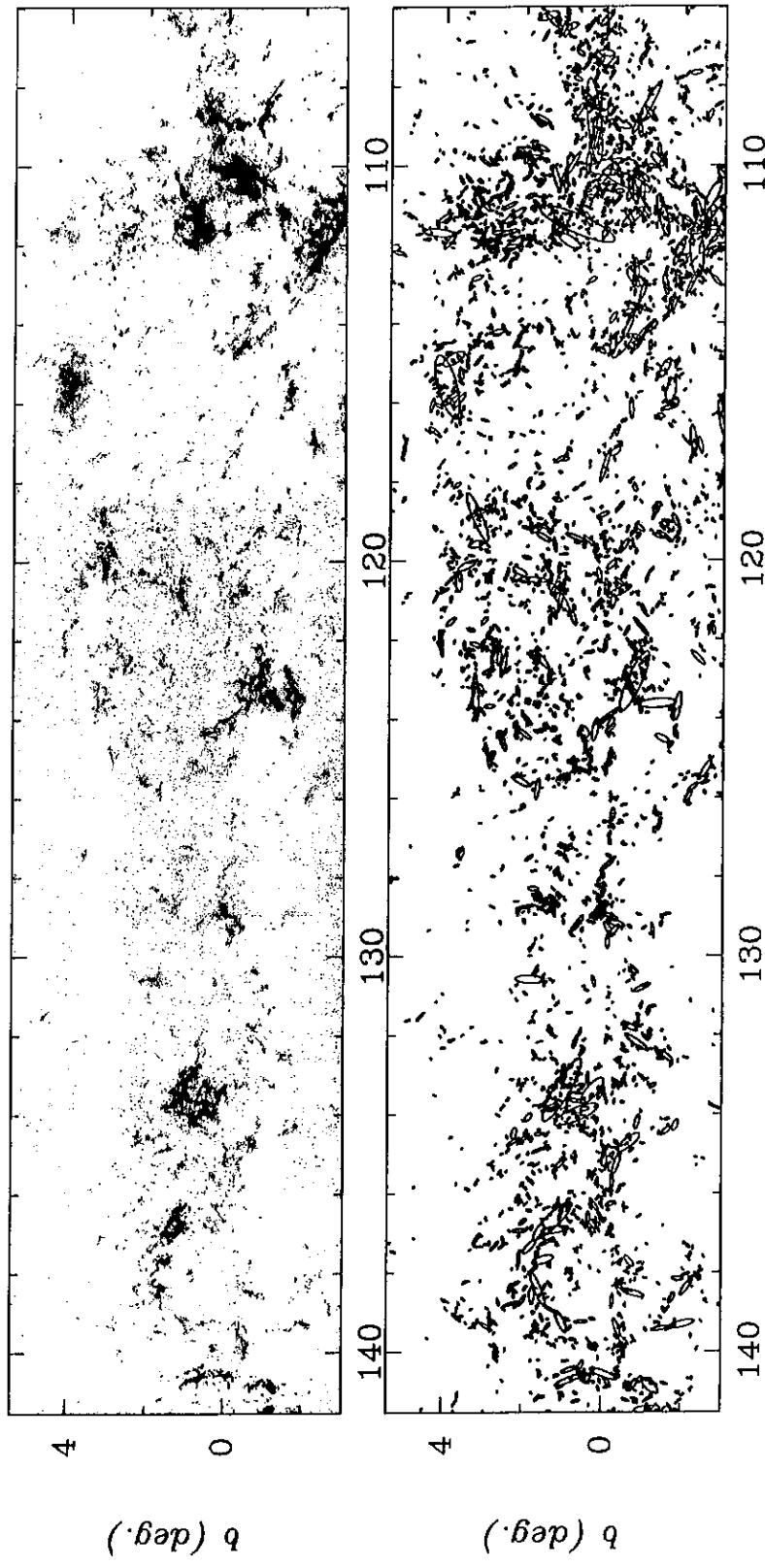


Fig. 1.—Top: Image of $^{12}\text{CO} J = 1-0$ integrated intensity over the velocity interval -110 to -20 km s^{-1} from the FCRAO CO Survey of the outer Galaxy (Heyer et al. 1998). The half-tone ranges from 0 (white) to 20 (black) K km s^{-1} . Bottom: Positions, sizes, and orientations of identified objects approximated as ellipses.

$$N(H_2) = 1.9 \times 10^{20} L_{CO} (\text{cm}^{-2})$$

$\approx X_{CO}$

$$M_{CO} = 4.1 L_{CO} (M_\odot)$$

$\approx \text{K km/s pc}^2$

gravitational parameter

$$\alpha_G \approx \frac{5 \sigma_v^2 r_e}{G M}$$

$\alpha_G \approx 1 \rightarrow \text{self-gravitational}$

if clouds are not gravitationally bound,
then formula overestimates M

can show

$$X_{CO} = 4.1 \left(\frac{\langle T \rangle}{5 \kappa} \right)^{-1} \left(\frac{\langle n \rangle}{100 \text{ cm}^{-3}} \right)^{1/2} \alpha_G^{-1/2}$$

[not gravitationally bound $\rightarrow \alpha_G \gg 1$]

$$\frac{\Delta N}{\Delta L_{CO}} = (3.1 \times 10^4) L_{CO}^{-1.80} \rightarrow 50\% \text{ from clouds}$$

with $L_{CO} > 7800 \text{ K km/s pc}^2$

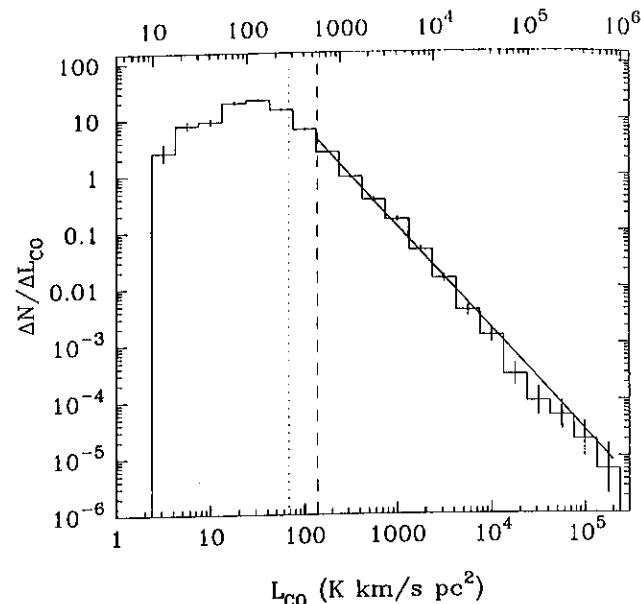


FIG. 3.—CO luminosity function, $\Delta N/\Delta L_{CO}$, for 3901 identified objects. The top x-coordinate shows the corresponding mass scales assuming a constant CO-to-H₂ conversion factor. The vertical dotted line denotes the detection limit of L_{CO} , and the vertical dashed line marks the completeness limit of L_{CO} at a distance of 10 kpc. The power-law fit to bins above the completion limit (solid line) is $\Delta N/\Delta L_{CO} \propto L_{CO}^{-1.80 \pm 0.03}$.

velocity dispersion σ_v

→ thermal

→ turbulent

→ rotational

→ expanding motions

σ_v increases with r_e

for $r_e > 7 \text{ pc}$

BUT no trend for

$r_e < 7 \text{ pc}$

$$M_{\text{vir}} = \frac{5 \sigma_v^2 r_e}{G}$$

$$\alpha_G = \frac{M_{\text{vir}}}{M_{\text{CO}}} = 53 L_{\text{CO}}^{-0.5}$$

- only most luminous clouds have $\alpha_G \sim 1$

⇒ smaller clouds
are internally
overpressed
with respect to gravity

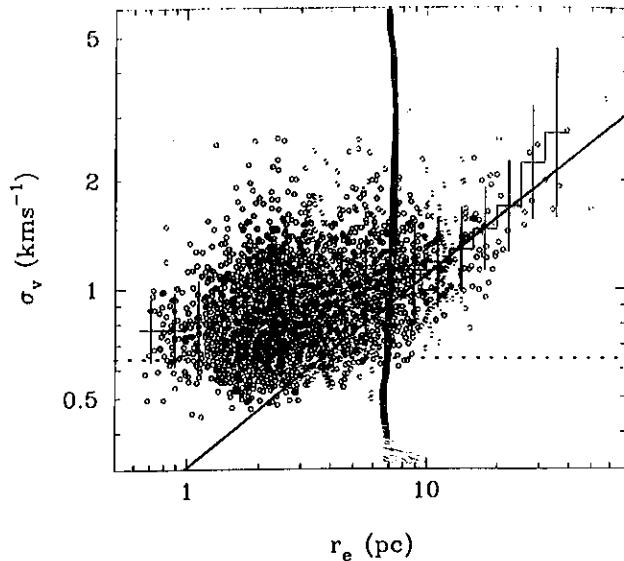


FIG. 6.—Variation of measured velocity dispersion, σ_v , and effective size, r_e . The thin solid line shows the mean value within logarithmic bins of r_e , and the error bars reflect the dispersion of values about the mean in each bin. The thick solid line shows the power-law fit to the clouds with $r_e > 9 \text{ pc}$. The slope of the power law is similar to that found by Solomon et al. (1987). The horizontal dashed line shows the velocity dispersion to which the measured values are accurate to within 15%.

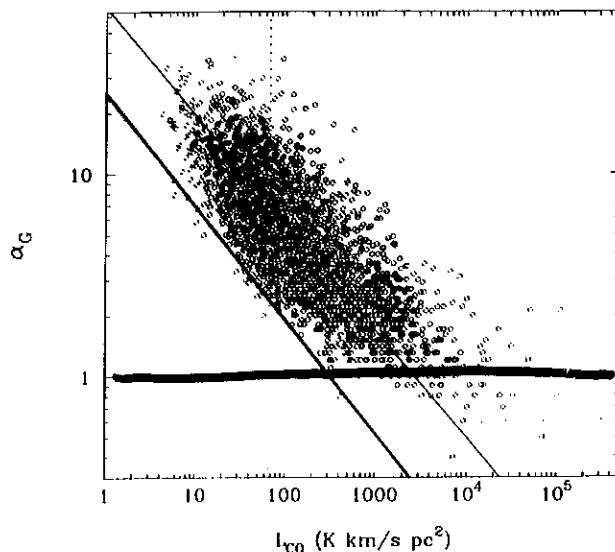


FIG. 7.—Variation of the gravitational parameter, α_G , with CO luminosity. The dotted vertical line denotes the detection limit of L_{CO} at a distance of 10 kpc. The thick solid line shows the minimum value of α_G to which the decomposition is sensitive as a result of the observational selection effect that excludes narrow-line clouds.

regions with large σ_v^2/l
must be either

- short lived, or
- bound by external pressure

$$t_{\text{dyn}} \sim \frac{l_{\text{min}}}{2\sigma_v} \sim 5-50 \times 10^4 \text{ yr}$$

$$\ll t_{\text{chem}} > 10^6 \text{ yr}$$

more likely pressure bounded

Virial theorem :

$$\frac{\sigma_v^2}{l_{\text{min}}} = \frac{P_0}{\kappa} \frac{k}{m_{H_2}} \frac{1}{N_{H_2}}$$

$$+ \frac{\pi G M_{H_2}}{5} N_{H_2}$$

for observed σ_v^2/l ,
mean pressure

$$\sim 1.4 \times 10^4 \text{ K/cm}^3$$

most likely source
of this pressure is weight of overlying
layer of atomic gas
need $N_H \sim 2 \times 10^{21} \text{ cm}^{-2}$ to shield H_2

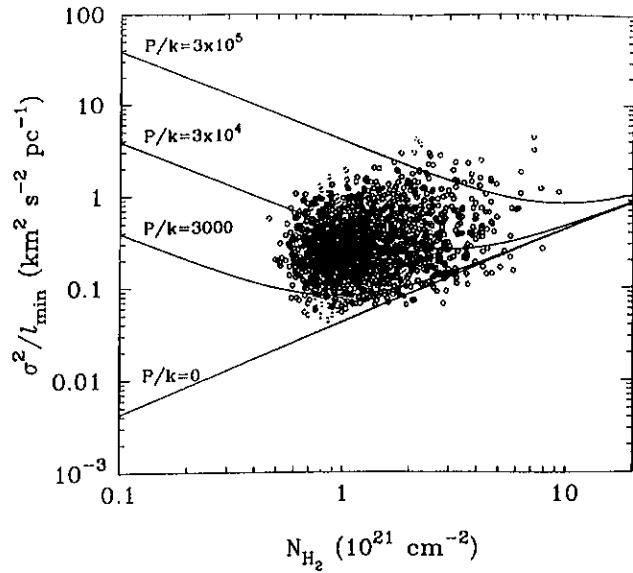


FIG. 11.—Values of $\sigma_v^2/l_{\text{min}}$ vs. the mean column density for the identified objects. The solid lines show the variation of this value for varying external pressures for bound objects.

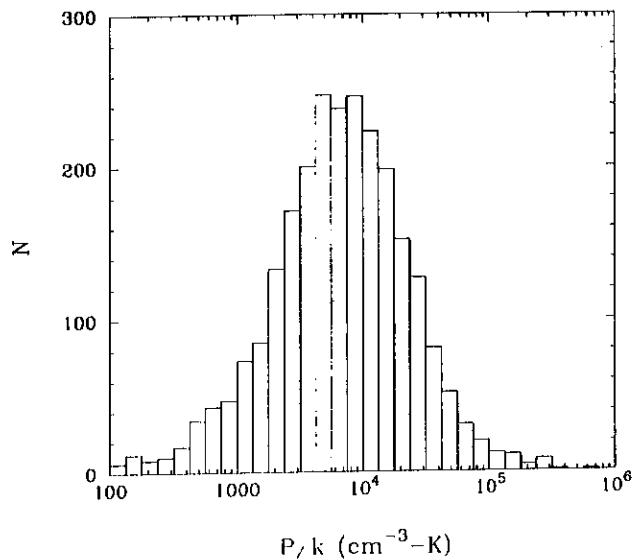


FIG. 12.—Distribution of required external pressures to bind the internal motions of identified molecular regions.

GMC Lifetimes : Still Unresolved

(1) long lifetimes ($> 10^8$ yr) (Scoville + Sanders 1987)

- substantial numbers of GMCs in interarm regions
 - lifetimes longer than time between spiral arm passages
- also Z scale-height decreases with increasing cloud mass
 - equipartition of GMC kinetic energy
 - survive several cloud-cloud collision times

(2) short lifetimes (few $\times 10^7$ yr) (Elmegreen 1985 PPII)

- from star cluster ages + cloud statistics

$$\langle t_{\text{cloud}} \rangle = t_{\text{cluster}} \left(1 + \frac{N_{\text{with cluster}}}{N_{\text{without cluster}}} \right)$$

- ages of OB associations

$$\sim 10^7 \text{ yr}$$

$$\sim 1$$

GMC lifetimes: approaching consensus?

- ① short estimates ~ 10 Myr (ie Elmegreen 2000, ApJ)
- based on age spread of young stars ie $\lesssim 10$ Myr for OB association
- \rightarrow star formation in $1-2 \times t_{\text{cross}} = \frac{R}{\sigma_v}$

- ② long estimates ~ 100 Myr (ie Scoville & Wilson 2004, conf. proceedings)
- mass conservation on Galactic orbit implies

$$\frac{M_{\text{H}_2}}{\tau_{\text{H}_2}} = \frac{M_{\text{H}_I}}{\tau_{\text{H}_I}} \quad M_{\text{H}_2} > M_{\text{H}_I} \text{ in inner disk}$$

τ_{H_I} = smaller of

$\begin{cases} \text{arm to arm transit time} \\ t_{\text{dynamical}} \quad (n \sim 10 \text{ cm}^{-3}) \end{cases}$
--

$$\Rightarrow \tau_{\text{H}_I} > 5 \times 10^7 \text{ yr} \quad \Rightarrow \tau_{\text{H}_2} \gtrsim 10^8 \text{ yr}$$

(Note $\tau_{\text{H}_2} \neq$ GMC lifetime necessarily)

- ③ lifetimes ~ 30 Myr from theory (Williams & McKee 1997) and observations (Engargiola et al. 2003; Rosolowsky 2005, Ph.D.)

Theory: $10^6 M_\odot$ cloud destroyed in 30 Myr by photoevaporation by O stars

Observations: association with HI filaments, HII regions

Why are the column densities of GMCs roughly constant?

4. theories

1. N is near but below critical value for support by \vec{B} fields (Shu et al ARAA 25, 23 (1987))
2. Clouds are in equilibrium with external pressure (Elmegreen 1989 ApJ 338, 178)

$$P_{\text{ext}} \sim \frac{GM^2}{R^4} \propto N^2$$

3. "Photoionization regulated star formation"
(McKee 1989 ApJ 345, 782)
 - time scale for ambipolar diffusion $\propto X_e$
 - low mass star formation only possible if X_e low
 - diffuse cloud contracts quickly to $A_V \sim 4$, central parts shield, + ad drops, star formation starts
 - outflows stabilize cloud \rightarrow ^{column densities}_{of most clouds} _{near onset value}

4. No physical basis \rightarrow due to observational bias & sensitivity limits eg Kegel 1989 AA 225, 517